The quantum fate of black hole horizons

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With Clément Berthiere and Deb Sarkar; arXiv:1712.09914

Outline of the talk

- Motivations: wormholes as black hole mimickers
- Properties of classical black hole horizons
- Semiclassical gravity
- Black holes or wormholes in semiclassical gravity
- Conclusions

Motivation

Perturbations of classical black holes:

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Perturbations of classical black holes:

- have continuous spectrum
- have complex poles (QNM)
- relaxation back to equilibrium is due to exponential decay

On the other hand, black holes have finite entropy!

As any (classical or quantum) system of finite entropy they should show Poincaré recurrences

$$t_{Poincare} \sim e^{S_{BH}}$$

Susskind et al. '02

The source of this discrepancy is infinite volume in optical metric

$$ds^2 = g(r)dt^2 + g(r)^{-1}dr^2 + r^2 d\omega_d^2 = g(r)ds_{opt}^2 \,,$$

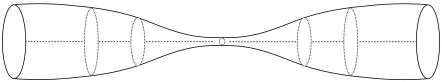
$$g(r) = 1 - r_+/r \,, \quad 0 \le t \le \beta_H$$

$$V_{opt} = 4\pi\beta_H \int \frac{drr^4}{(r-r_+)^2} \to \infty \; if \; r \to r_+$$

A smooth way to regularize this divergence is to replace black hole with a wormhole

$$g_{tt} \to g_{tt} + \lambda^2 \,, \quad \lambda^2 \ll 1$$

$$ds^2_{wh} = (g(r) + \lambda^2)dt^2 + g(r)^{-1}dr^2 + r^2 d\omega^2_d$$



New properties:

- there is no event horizon
- instead there is a throat at $r = r_+$ of size $L \sim r_+ \ln 1/\lambda$
- $t_{throat} \sim \lambda t_\infty$
- two new time scales:

$$t_{Heisenberg} \sim \ln 1/\lambda$$

$$t_{Poincare} \sim 1/\lambda$$

If $\lambda \sim e^{-S_{BH}}$ one has a realization of Susskind's ideas

Important:

during time scales $\ll t_{Heisenberg}, t_{Poincare}$ no difference with true black holes

S.S.'04, '05; T. Damour and S.S.'07

Applications in astrophysics/ gravitational waves:

Wormholes of this type are considered as exotic compact object (ECOs) that may produce same gravitational wave signals as black holes

Many papers including Cardoso, Franzin and Pani '16;

Bueno, Cano, Goelen, Hertog and Vernocke '17

So far our wormhole was considered as a phenomenological metric.

We obtain it as a solution to equations of semiclassical gravity.

Universality at horizon

$$ds^2 = g(r)dt^2 + e^{2\phi(r)}g^{-1}(r)dr^2 + r^2 d\omega_d^2$$

$$g(r) = \frac{4\pi}{\beta}(r - r_+) + O(r - r_+)^2\,, \quad \phi(r) = O(r - r_+)$$

Optical metric

$$ds^2 = g(z)ds^2_{opt}\,, \quad ds^2_{opt} = dt^2 + dz^2 + R^2(z)d\omega_d^2$$

$$g(z) \sim e^{-4\pi z/\beta} + \ldots\,, \quad R^2(z) \sim e^{4\pi z/\beta} + \ldots \quad z \to \infty$$

Optical spacetime is product space $S_1^\beta \times M_3$

Near horizon M_3 is hyperbolic space H_3 of radius $\beta/2\pi$

It is a solution to GR equations to leading order for any β

Horizon as a minimal surface

$$ds^2 = \Omega^2(\rho)dt^2 + d\rho^2 + r^2(\rho)d\omega_d^2$$

$\Omega^2 = g$ and ρ is geodesic radial coordinate

Einstein equations:

$$2rr'' + r'^2 - 1 = 0$$

$$\Omega(r'^2 - 1) + 2rr'\Omega' = 0$$

If 2-sphere at $\rho = \rho_+$ has minimal area $r'(\rho_+) = 0$ then $\Omega(\rho_+) = 0$ and this sphere is a horizon

Goal of this talk

Study whether same properties are valid in semiclassical gravity (SG)

Claims

- static spherically symmetric metric with a horizon of *finite* (non-vanishing) temperature is not a solution to SG
- in SG a 2-sphere of minimal area embedded in static space-time is not a horizon. Instead it is a throat of a wormhole
- Ω^2 at throat is bounded by $e^{-S_{BH}}$ (consistent with Susskind's ideas)
- Possible temperature is different from Hawking temperature and is exponentially small

Before we start: general form of 4-metric we shall consider

$$ds^2 = \Omega^2(z)\left(dt^2 + N^2(z)dz^2 + R^2(z)d\omega_2^2\right)\,, \quad \Omega(z) = e^{\sigma(z)}$$

Useful choices of coordinates (gauge fixing):

- Black hole horizon

$$N(z) = 1\,, \quad \sigma(z) = -2\pi z/\beta + \dots\,, \quad R(z) \simeq r_+ e^{2\pi z/\beta}$$

  apriori β and r_+ are not related

- Minimal sphere

$$N(z) = 1/\Omega(\rho)\,, \quad z = \rho\,, \quad R(\rho) = r(\rho)/\Omega(\rho)$$

  $r(\rho)$ is geometric radius of sphere

Semiclassical Gravity (SG)

- Consider backreaction from quantized scalar, gauge and fermion fields on non-quantized geometric background.

- Non-perturbative handle is due to the study of conformal anomaly.

  Fradkin-Tseytlin '84, Dowker-Schofield '90, Mazur-Motola '01

- Two important contributions: due to anomaly and due to optical metric

Gravitational action

$$W_{grav} = -\frac{1}{16\pi G_N}\int R[G] + \Gamma[G]$$

$\Gamma[G]$ is quantum effective action, result of integrating out quantum fields

we represent $G_{\mu\nu} = e^{2\sigma} g_{\mu\nu}$, quantum effective action transforms as

$$\Gamma[e^{2\sigma}g] = -\frac{a}{16\pi^2}\int \sigma C^2 + \frac{b}{16\pi^2}\int \sigma E - \frac{2b}{16\pi^2}\int (2\tilde{G}^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma + 2\Box\sigma(\nabla\sigma)^2 + (\nabla\sigma)^4) + \Gamma_0[g]$$

$\tilde{G}^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R$ is Einstein tensor

$$C^2 = Riem^2 - 2Ricci^2 + \frac{1}{3}R^2\,, \quad E = Riem^2 - 4Ricci^2 + R^2$$

$$a = \frac{1}{120}(n_0 + 6n_{1/2} + 12n_1)\,, \quad b = \frac{1}{360}(n_0 + 11n_{1/2} + 62n_1)$$

Effective action on optical metric:

$$\Gamma_0 = \Gamma[S_1^\beta \times M_3] = -\frac{\pi^2}{90\beta^3} c_H \int_{M_3} 1 + \frac{\lambda_H}{144\beta} \int_{M_3} R_{M_3}$$

$$c_H = n_0 + \frac{7}{2} n_{1/2} + 2n_1 \, , \quad \lambda_H = n_{1/2} + 4n_1$$

- exact result if $M_3 = H_3$
- we dropped higher curvature (non-local) terms
- general structure discussed by Gusev and Zelnikov '98

Horizons in SG

Variations of $W_{grav}[\sigma(z), N(z), R(z)]$ w.r.t. $\sigma(z)$, $N(z)$ and $R(z)$ give semiclassical gravitational equations

Some observations: $E(g_{optical}) = 0$ and $C^2(g_{optical}) \to C^2(S_1 \times H_3) = 0$ as $z \to \infty$

Variation w.r.t. $N(z)$ will produce divergent (as $z \to \infty$) terms. These terms will come from derivatives of σ in b-anomaly and from Γ_0, the divergence is due to divergent volume density on M_3

$$
\begin{aligned}
\delta_N W_{grav} &= (360b - 2c_H - 10\lambda_H)\, \frac{\pi^2}{180\beta^4}\, r_+^2 e^{4\pi z/\beta} \\
&= -\left(n_0 + 6n_{1/2} - 18n_1\right) \frac{\pi^2}{180\beta^4}\, r_+^2 e^{4\pi z/\beta} .
\end{aligned}
$$

- Curiously, the equations are satisfied for $\mathcal{N} = 4$ SYM theory (have to look at subleading terms)

- For generic set of fields divergent term is there so that no static solutions with horizons in SG!

Minimal sphere

We look for solutions with a turning point: $r'(\rho) = 0$ and $\Omega'(\rho) = 0$ at $\rho = \rho_+$ such that $r'' > 0$ and $\Omega'' > 0$

Such a solution is parametrized by values of r and Ω at turning point

r is the radius of classical horizon

Values of second derivatives r'' and Ω'' are determined by r and Ω via gravitational equations

Additionally, there arise consistency conditions on possible values of Ω provided r can be arbitrary

Variation w.r.t. $N(z)$ takes the form at the turning point

$$(\Omega r r'' - r^2\Omega'')^2 = y^2\Omega^2\,, \quad y^2 = 1 - \frac{r^2}{\kappa\bar{a}\ln\Omega^{-1}}\left(\frac{\gamma\kappa r^2}{\beta^4\Omega^4} - \frac{\lambda\kappa}{\beta^2\Omega^2} + 1\right)$$

$\kappa = 8\pi G_N$, $\bar{a} = a/12\pi^2$, $\gamma = c_H\pi^2/90$, $\lambda = \lambda_H/72$

Condition that $y^2 \geq 0$ restricts possible values of Ω!

For simplicity consider $\lambda_H = 0$ (only scalars)

Then condition $y^2 \geq 0$ is equivalent to condition

$$\Omega^4 \ln \frac{\Omega_0}{\Omega} \geq \frac{\gamma r^4}{\bar{a}\beta^4}\, , \quad \Omega_0 = e^{-\frac{r^2}{\kappa \bar{a}}}$$

Notice that $\frac{r^2}{\kappa \bar{a}}$ is proportional to Bekenstein-Hawking entropy $S_{BH} = 8\pi^2 r^2/\kappa$ of classical black hole

It immediately follows that

- $\Omega < \Omega_0 = e^{-\frac{r^2}{\kappa \bar{a}}}$
- $T^4 = 1/\beta^4 < \frac{\bar{a}}{4\gamma r^4}\Omega_0^4$, i.e. temperature is exponentially small!

Conditions $r'' > 0$ and $\Omega'' > 0$ impose extra constraints on possible values of Ω.

In classical limit $\bar{a} \to 0$ so that $\Omega_0 = 0$ and the throat becomes horizon!

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$$t_{distinguish} \sim GM \log 1/\Omega_0 \sim G^2 M^3$$

Damour, Solodukhin '07

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- Bound on temperature
- Experimental signatures for early universe black holes. Lower temperature with longer life span.

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- Wormhole modifications provide an interesting resolution for information problem.
- Understanding small BHs.
- Possible implications for primordial black holes and dark matter.

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