

# Modified Gravity before and after GW170817

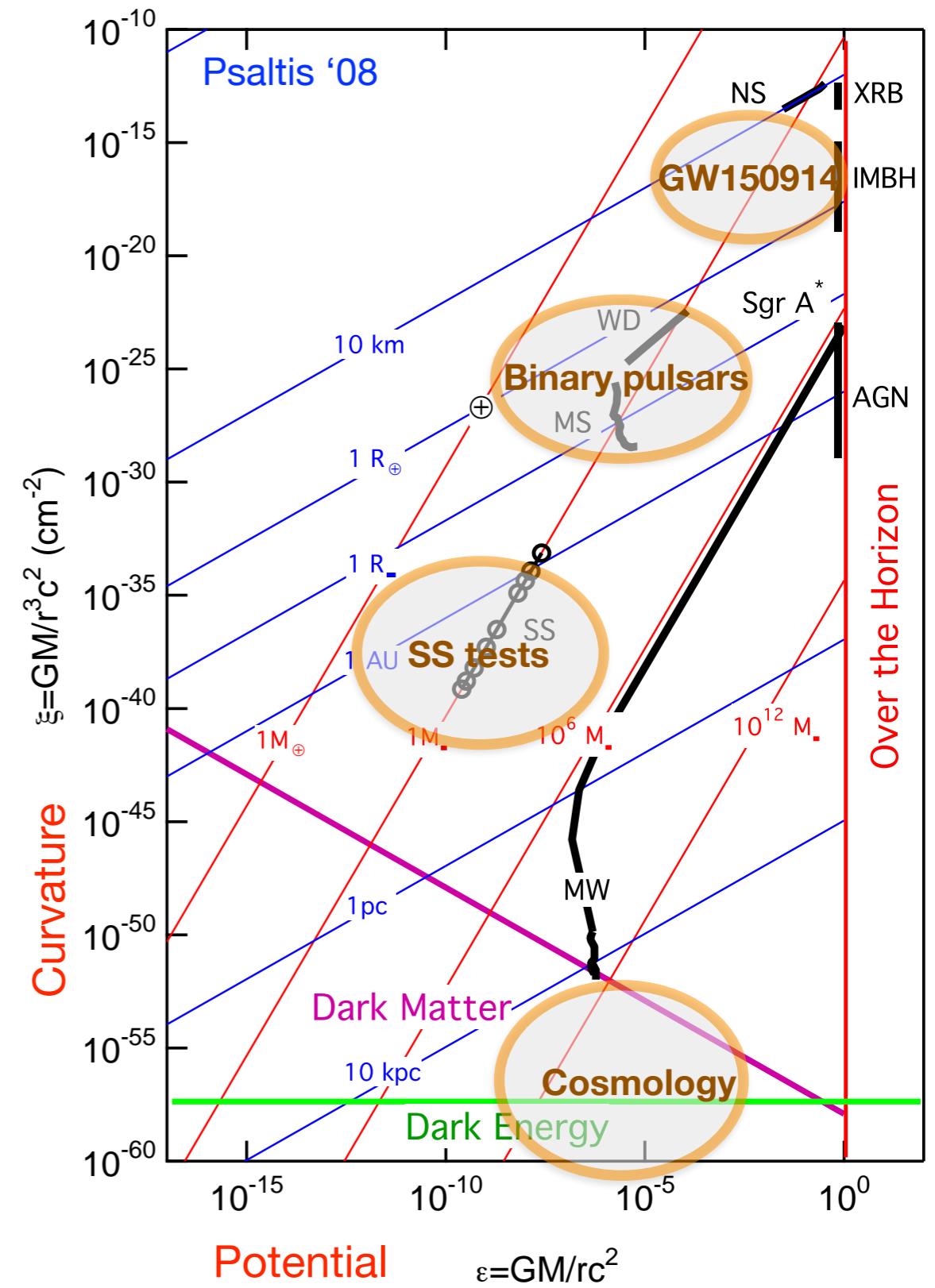
Filippo Vernizzi - IPhT, CEA Saclay

RTG workshop Bremen “Models of gravity”, 19 February 2018

# Gravity on large scales

Acceleration of the Universe has generated a lot of activity in trying to extend cosmology and gravity beyond standard LCDM model.

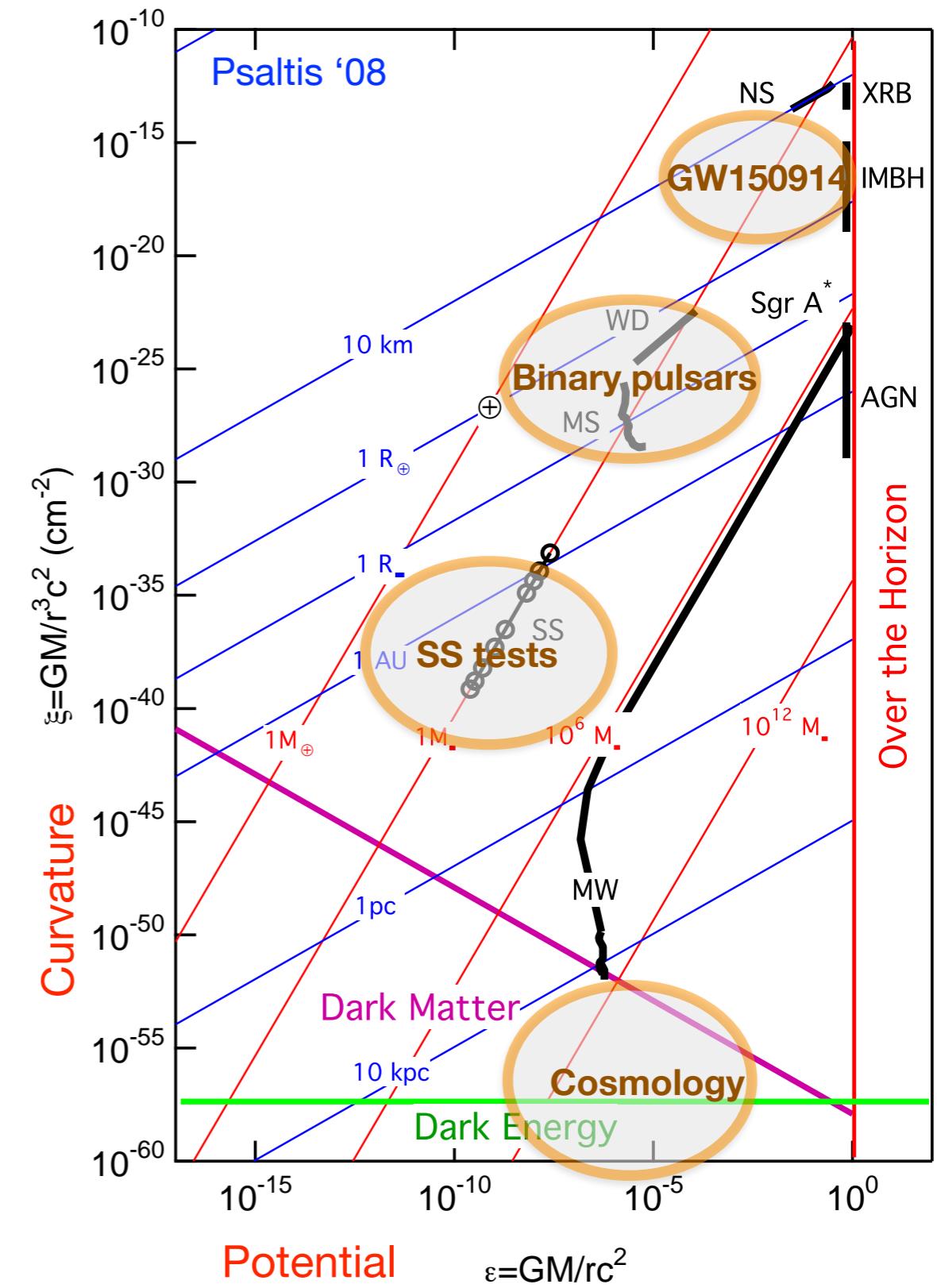
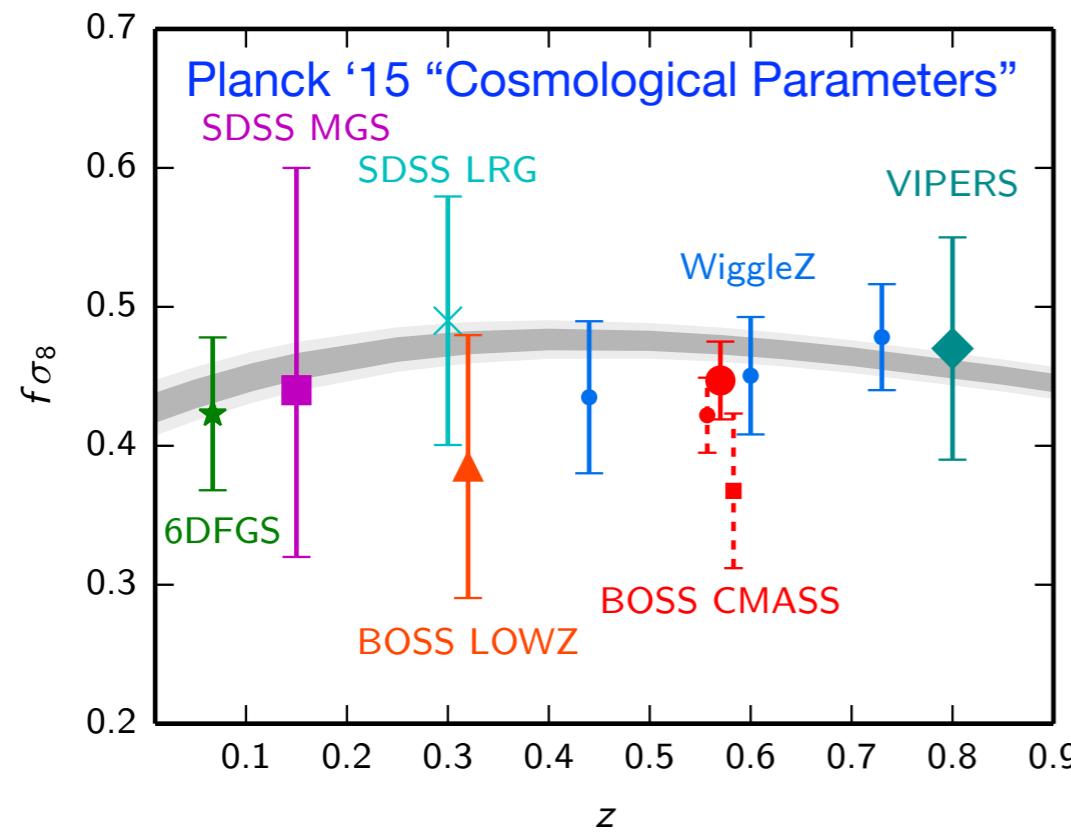
GR tested only on special range of scales and masses.



# Gravity on large scales

Acceleration of the Universe has generated a lot of activity in trying to extend cosmology and gravity beyond standard LCDM model.

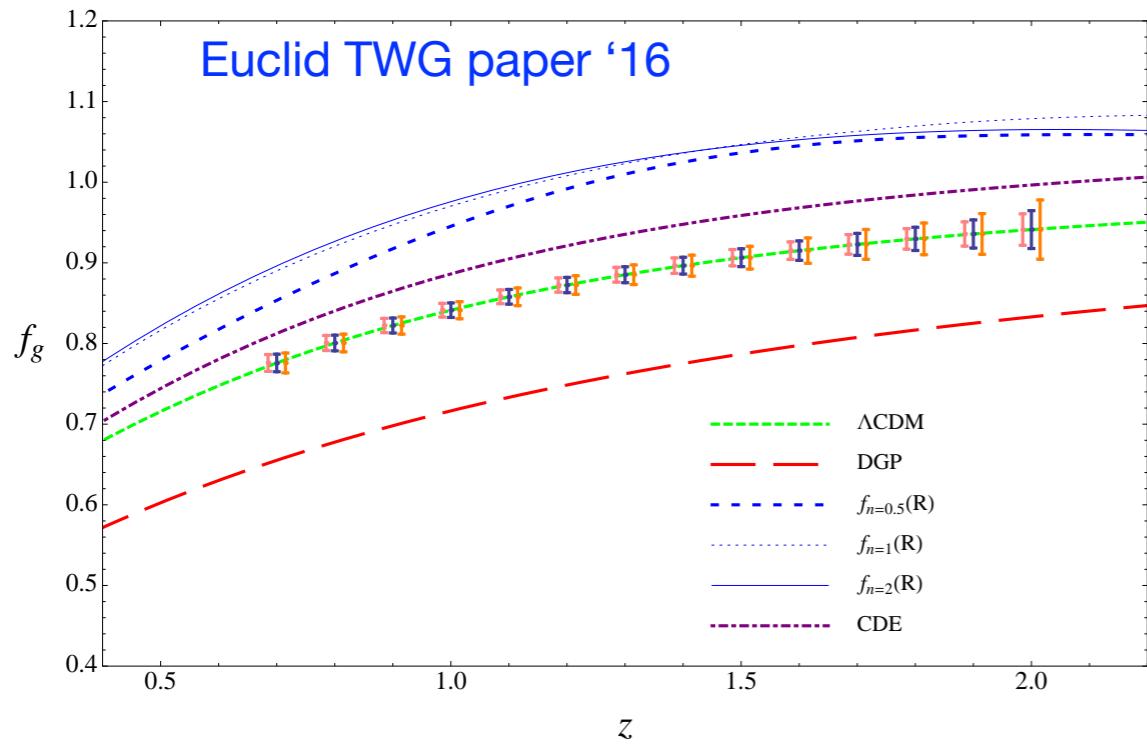
GR tested only on special range of scales and masses.



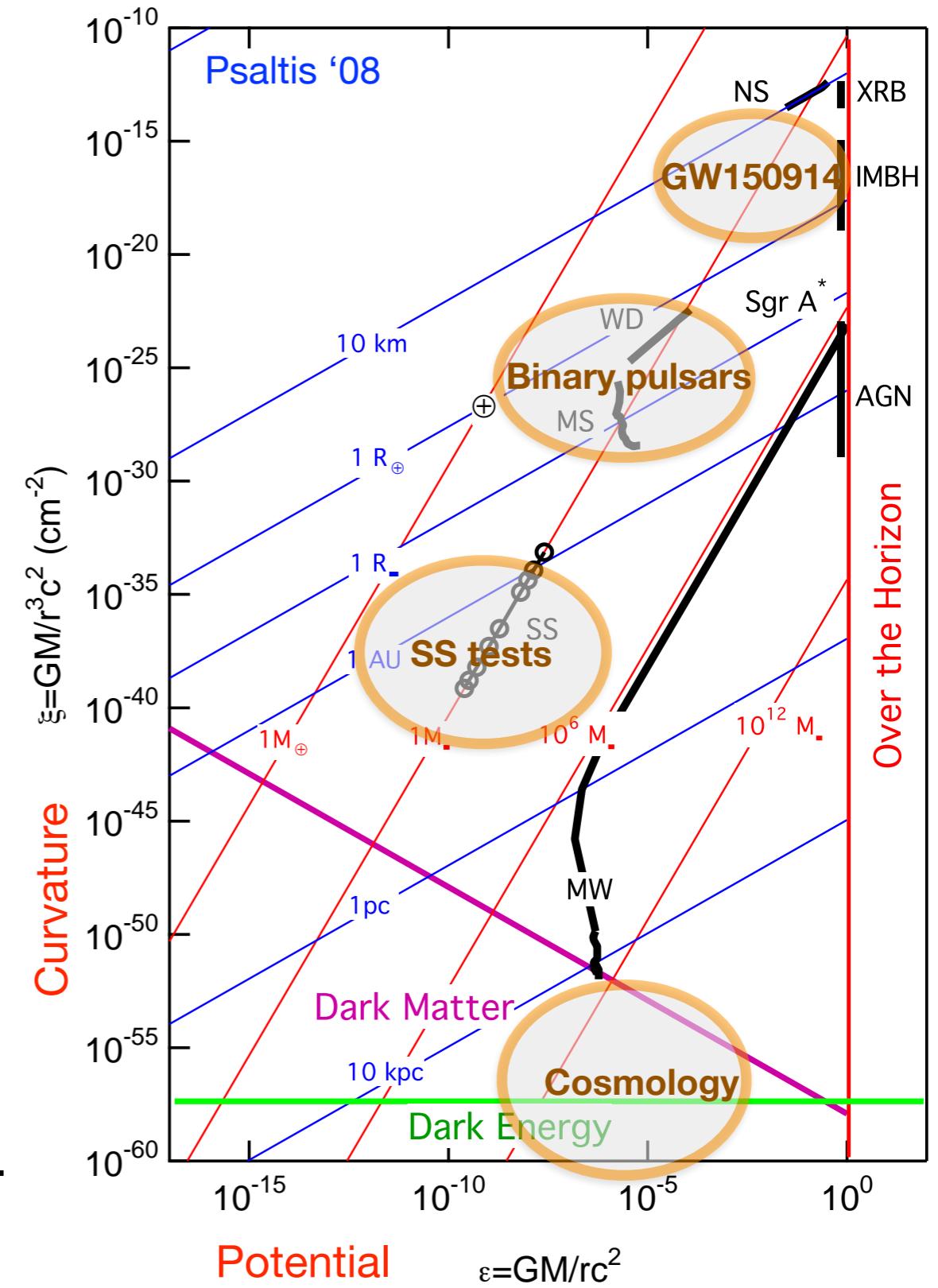
# Gravity on large scales

Acceleration of the Universe has generated a lot of activity in trying to extend cosmology and gravity beyond standard LCDM model.

GR tested only on special range of scales and masses.



Cosmology window for discovery of new physics.  
Analogous to precision tests at LHC



# Scalar-tensor theories

- ◆ Simplest models of modified gravity are base on single scalar field
- ◆ Old school theories: Quintessence, Brans-Dicke, K-essence, ...  $\mathcal{L}(\phi, \partial_\mu \phi)$

# Scalar-tensor theories

- ◆ Simplest models of modified gravity are base on single scalar field
- ◆ Old school theories: Quintessence, Brans-Dicke, K-essence, ...       $\mathcal{L}(\phi, \partial_\mu \phi)$

Ex:      
$$\mathcal{L} = R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$

$$\mathcal{L} = f(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$

$$\mathcal{L} = R + G_2(\phi, X) , \quad X \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

# Scalar-tensor theories

- ◆ Simplest models of modified gravity are base on single scalar field
- ◆ Old school theories: Quintessence, Brans-Dicke, K-essence, ...  $\mathcal{L}(\phi, \partial_\mu \phi)$
- ◆ Generalized theories: Galileons  $\mathcal{L}(\phi, \partial_\mu \phi, \nabla_\mu \nabla_\nu \phi)$

Ex: Massive gravity  $g_{\mu\nu} \supset \partial_\mu \partial_\nu \phi$  longitudinal mode of 5D graviton

DGP model  $g_{5\mu} \supset \partial_\mu \phi$  brane-bending mode

# Self-acceleration and fifth force

- ◆ Simplest models of modified gravity are base on single scalar field
- ◆ Old school theories: Quintessence, Brans-Dicke, K-essence, ...  $\mathcal{L}(\phi, \partial_\mu \phi)$
- ◆ Generalized theories: Galileons  $\mathcal{L}(\phi, \partial_\mu \phi, \nabla_\mu \nabla_\nu \phi)$

Can provide self-acceleration and nonlinearities (Vainshtein screening),  
with controlled quantum corrections and no ghost

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \frac{\alpha}{M_{\text{Pl}}}\phi T_m$$



# Self-acceleration and fifth force

- ◆ Simplest models of modified gravity are base on single scalar field
- ◆ Old school theories: Quintessence, Brans-Dicke, K-essence, ...  $\mathcal{L}(\phi, \partial_\mu \phi)$
- ◆ Generalized theories: Galileons  $\mathcal{L}(\phi, \partial_\mu \phi, \nabla_\mu \nabla_\nu \phi)$

Can provide self-acceleration and nonlinearities (Vainshtein screening), with controlled quantum corrections and no ghost

$$\mathcal{L} = -\frac{1}{2} \left( 1 + \frac{\square \phi}{\Lambda_3^3} \right) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) + \frac{\alpha}{M_{\text{Pl}}} \phi T_m$$



$$\frac{\square \phi}{\Lambda_3^3} \gg 1$$

Vainshtein screening: large classical scalar field nonlinearities

# Scalar and gravity playing

- ◆ Simplest models of modified gravity are base on single scalar field
- ◆ Old school theories: Quintessence, Brans-Dicke, K-essence, ...  $\mathcal{L}(\phi, \partial_\mu \phi)$
- ◆ Generalized theories: Galileons  $\mathcal{L}(\phi, \partial_\mu \phi, \nabla_\mu \nabla_\nu \phi)$

Scalar field and gravity play through these terms:

$$\nabla_\mu \nabla_\nu \phi \supset \Gamma_{\mu\nu}^\rho \partial_\rho \phi$$

$$\Gamma_{ij}^0 \dot{\phi} \supset \dot{\gamma}_{ij} \dot{\phi}$$



$$\mathcal{L}_\gamma \sim (\dot{\gamma}_{ij})^2 - c_T^2 (\partial_k \gamma_{ij})^2$$



# Horndeski theories

Covariantization of the Galileons. Most general Lorentz-invariant scalar-tensor theories with 2nd-order EOM. No extra modes: 1 scalar + 2 tensor polarisations

Horndeski 73, Deffayet et al. 11

$$\mathcal{L}_H^{(2)} = G_2(\phi, X)$$

$$X = \nabla_\mu \phi \nabla^\mu \phi$$

$$\mathcal{L}_H^{(3)} = G_3(\phi, X) \square \phi$$

$$\square \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$$

$$\mathcal{L}_H^{(4)} = G_4(\phi, X) R - 2G_{4,X}(\phi, X) [(\square\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2]$$

$$\mathcal{L}_H^{(5)} = G_5(\phi, X) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi + \frac{1}{3} G_{5,X}(\phi, X) [(\square\phi)^3 - 3\square\phi(\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3]$$

# Horndeski theories

Covariantization of the Galileons. Most general Lorentz-invariant scalar-tensor theories with 2nd-order EOM. No extra modes: 1 scalar + 2 tensor polarisations

Horndeski 73, Deffayet et al. 11

$$\mathcal{L}_H^{(2)} = G_2(\phi, X)$$

$$X = \nabla_\mu \phi \nabla^\mu \phi$$

$$\mathcal{L}_H^{(3)} = G_3(\phi, X) \square \phi$$

$$\square \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$$

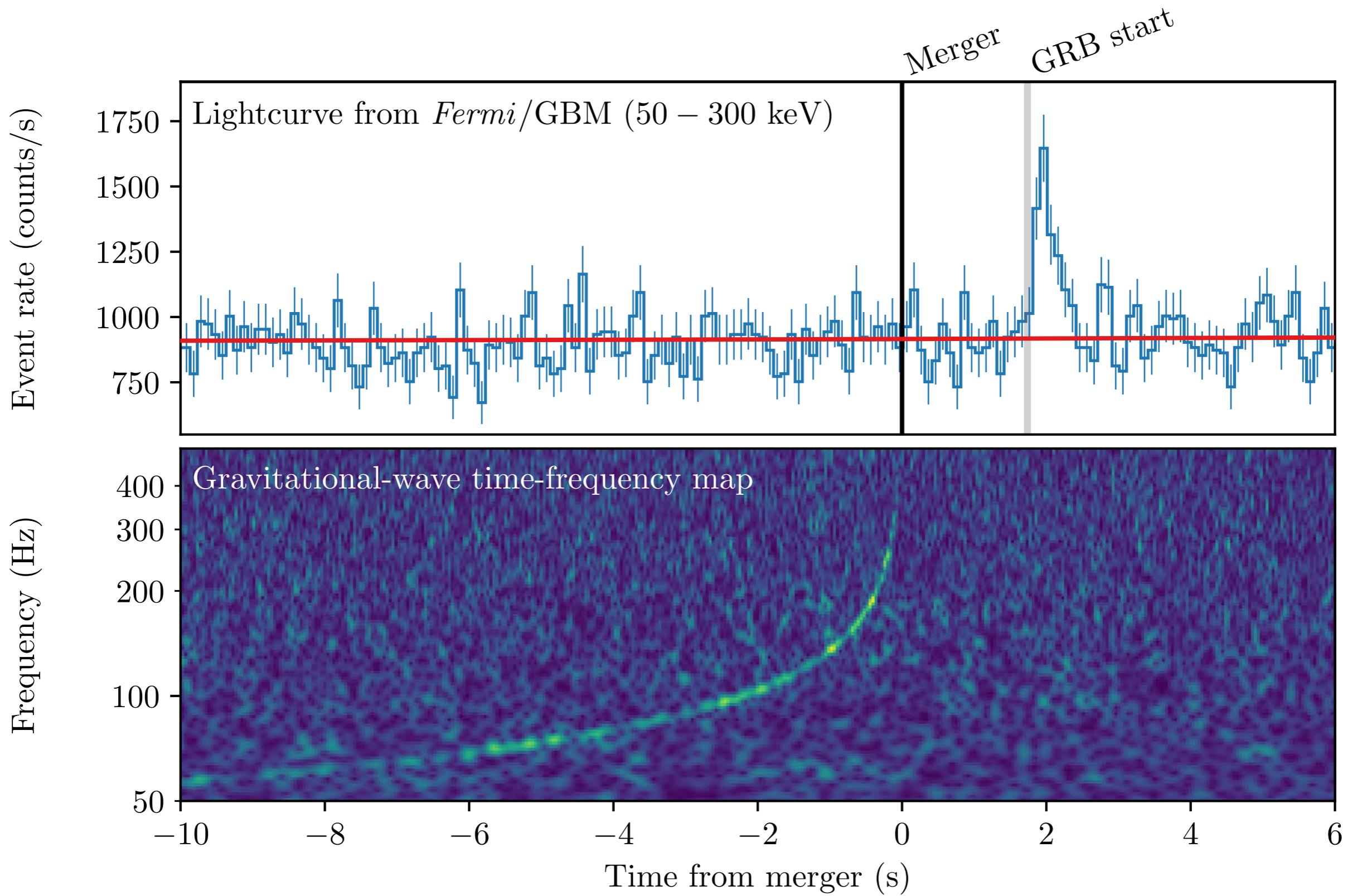
$$\mathcal{L}_H^{(4)} = G_4(\phi, X) R - 2G_{4,X}(\phi, X) [(\square\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2]$$

$$\mathcal{L}_H^{(5)} = G_5(\phi, X) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi + \frac{1}{3} G_{5,X}(\phi, X) [(\square\phi)^3 - 3\square\phi(\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3]$$

$$c_T^2 - 1 \propto -2G_{4,X} - G_{5,\phi} - (H\dot{\phi} - \ddot{\phi})G_{5,X}$$

- ♦ Regime of interest for LSS surveys:  $c_T^2 - 1 \sim \mathcal{O}(10^{-1}) - \mathcal{O}(10^{-2})$

# GW170817 = GRB170817



Virgo, LIGO Scient. coll. 17; Virgo, Fermi-GBM, INTEGRAL, LIGO Scient. coll. 17

# GW170817 = GRB170817

$$-3 \times 10^{-15} \leq \frac{\Delta\nu}{\nu_{\text{EM}}} \leq 7 \times 10^{-16}$$

- ◆ Previous limits on GW slower than light, Cherenkov radiation. One sided and at high energy  
[Moore and Nelson 01](#)
- ◆ Low energy GW:  $\lambda \sim 10\ 000$  Km. Can use cosmological description
- ◆ Over cosmological distance: 40 Mpc. Screening mechanism inefficient and anyway (probably) negligible

# Other coincident events

[2] [arXiv:1710.05901 \[pdf, other\]](#)

## **Dark Energy after GW170817**

[Jose María Ezquiaga](#) (1 and 2), [Miguel Zumalacárregui](#) (2 and 3) ((1) Madrid IFT, (2) UC Berkeley, (3) Nordita)

Comments: 9 pages, 3 figures

Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO); General Relativity and Quantum Cosmology (gr-qc); High Energy Physics -**

[3] [arXiv:1710.05893 \[pdf, other\]](#)

## **Implications of the Neutron Star Merger GW170817 for Cosmological Scalar-Tensor Theories**

[Jeremy Sakstein, Bhuvnesh Jain](#)

Comments: five pages, two figures

Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO); General Relativity and Quantum Cosmology (gr-qc); High Energy Physics -**

[4] [arXiv:1710.05877 \[pdf, ps, other\]](#)

## **Dark Energy after GW170817**

[Paolo Creminelli, Filippo Vernizzi](#)

Comments: 5 pages

Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO); General Relativity and Quantum Cosmology (gr-qc); High Energy Physics -**

[see also Baker et al. 17](#)

# Consequence of $c_T=1$ on Horndeski

Covariantization of the Galileons. Most general Lorentz-invariant scalar-tensor theories with 2nd-order EOM. No extra modes: 1 scalar + 2 tensor polarisations

$$\begin{aligned}\mathcal{L}_H^{(2)} &= G_2(\phi, X) & X &= \nabla_\mu \phi \nabla^\mu \phi \\ \mathcal{L}_H^{(3)} &= G_3(\phi, X) \square \phi & \square &\equiv g^{\mu\nu} \nabla_\mu \nabla_\nu \\ \mathcal{L}_H^{(4)} &= G_4(\phi, X) R - 2G_{4,X}(\phi, X) [(\square \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2] \\ \mathcal{L}_H^{(5)} &= G_5(\phi, X) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi + \frac{1}{3} G_{5,X}(\phi, X) [(\square \phi)^3 - 3\square \phi (\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3]\end{aligned}$$

$$c_T^2 - 1 \propto -2G_{4,X} - G_{5,\phi} - (H\dot{\phi} - \ddot{\phi})G_{5,X}$$

- ◆ We want  $c_T = c$  for any “background”, i.e. any  $\dot{\phi}(t)$ ,  $\ddot{\phi}(t)$ ,  $H(t)$

# Consequence of $c_T=1$ on Horndeski

Covariantization of the Galileons. Most general Lorentz-invariant scalar-tensor theories with 2nd-order EOM. No extra modes: 1 scalar + 2 tensor polarisations

$$\begin{aligned}\mathcal{L}_H^{(2)} &= G_2(\phi, X) & X &= \nabla_\mu \phi \nabla^\mu \phi \\ \mathcal{L}_H^{(3)} &= G_3(\phi, X) \square \phi & \square &\equiv g^{\mu\nu} \nabla_\mu \nabla_\nu \\ \mathcal{L}_H^{(4)} &= G_4(\phi, X) R - 2G_{4,X}(\phi, X) [(\square \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2] \\ \mathcal{L}_H^{(5)} &= G_5(\phi, X) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi + \frac{1}{3} G_{5,X}(\phi, X) [(\square \phi)^3 - 3\square \phi (\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3]\end{aligned}$$

$$c_T^2 - 1 \propto -2G_{4,X} - G_{5,\phi} - (H\dot{\phi} - \ddot{\phi})G_{5,X}$$

- ◆ We want  $c_T = c$  for any “background”, i.e. any  $\dot{\phi}(t)$ ,  $\ddot{\phi}(t)$ ,  $H(t)$

# Consequence of $c_T=1$ on Horndeski

Covariantization of the Galileons. Most general Lorentz-invariant scalar-tensor theories with 2nd-order EOM. No extra modes: 1 scalar + 2 tensor polarisations

$$\begin{aligned}\mathcal{L}_H^{(2)} &= G_2(\phi, X) & X &= \nabla_\mu \phi \nabla^\mu \phi \\ \mathcal{L}_H^{(3)} &= G_3(\phi, X) \square \phi & \square &\equiv g^{\mu\nu} \nabla_\mu \nabla_\nu \\ \mathcal{L}_H^{(4)} &= G_4(\phi, \cancel{X}) R - 2G_{4,X}(\phi, \cancel{X}) [(\square \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2] \\ \mathcal{L}_H^{(5)} &= G_5(\phi, X) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi + \frac{1}{3} G_{5,X}(\phi, X) [(\square \phi)^3 - 3\square \phi (\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3]\end{aligned}$$

$$c_T^2 - 1 \propto -2G_{4,X} - G_{5,\phi} - (H\dot{\phi} - \ddot{\phi})G_{5,X}$$

- ◆ We want  $c_T = c$  for any “background”, i.e. any  $\dot{\phi}(t)$ ,  $\ddot{\phi}(t)$ ,  $H(t)$

# Consequence of $c_T=1$ on Horndeski

Covariantization of the Galileons. Most general Lorentz-invariant scalar-tensor theories with 2nd-order EOM. No extra modes: 1 scalar + 2 tensor polarisations

$$\begin{aligned}\mathcal{L}_H^{(2)} &= G_2(\phi, X) & X &= \nabla_\mu \phi \nabla^\mu \phi \\ \mathcal{L}_H^{(3)} &= G_3(\phi, X) \square \phi & \square &\equiv g^{\mu\nu} \nabla_\mu \nabla_\nu \\ \mathcal{L}_H^{(4)} &= G_4(\phi, \cancel{X}) R - 2G_{4,X}(\phi, \cancel{X}) [(\square \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2] \\ \mathcal{L}_H^{(5)} &= G_5(\phi, X) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi + \frac{1}{3} G_{5,X}(\phi, X) [(\square \phi)^3 - 3\square \phi (\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3]\end{aligned}$$

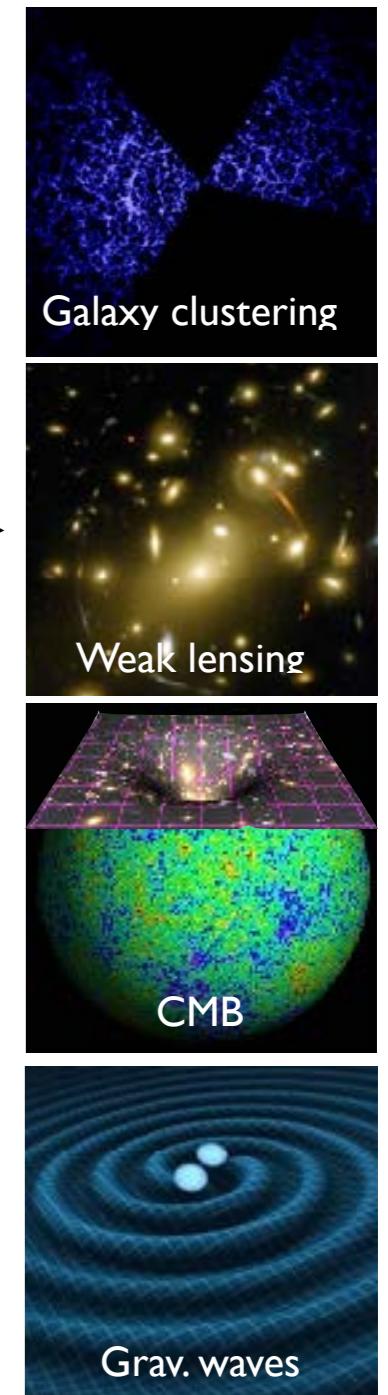
$$c_T^2 - 1 \propto -2G_{4,X} - G_{5,\phi} - (H\dot{\phi} - \ddot{\phi})G_{5,X}$$

- ♦ We want  $c_T = c$  for any “background”, i.e. any  $\dot{\phi}(t)$ ,  $\ddot{\phi}(t)$ ,  $H(t)$

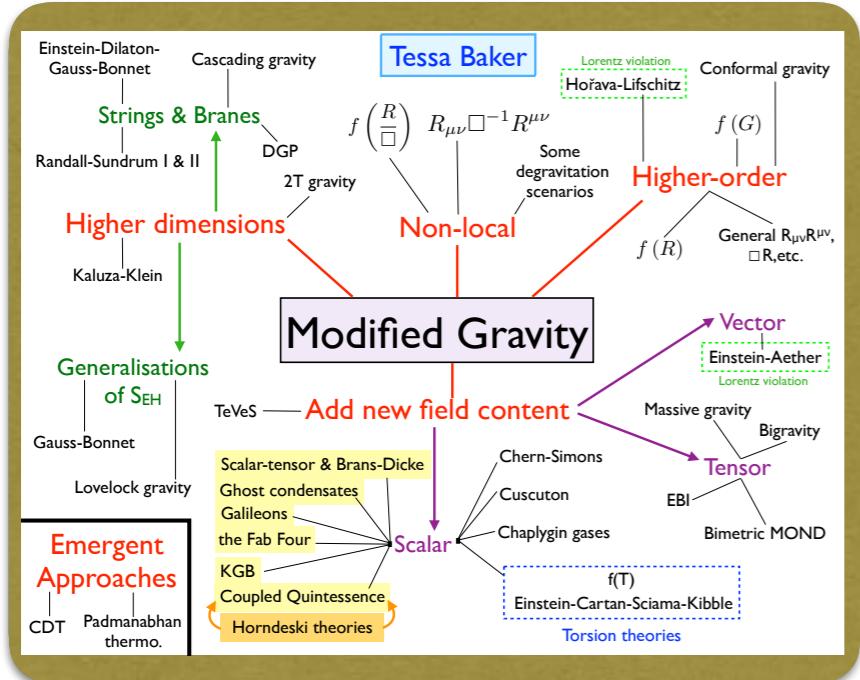
In conclusion:  $L_{c_T=1} = G_4(\phi)R + G_2(\phi, X) + G_3(\phi, X)\square\phi$

see also Ezquiaga, Zumalacarregui 17 and Baker et al. 17

# Observations



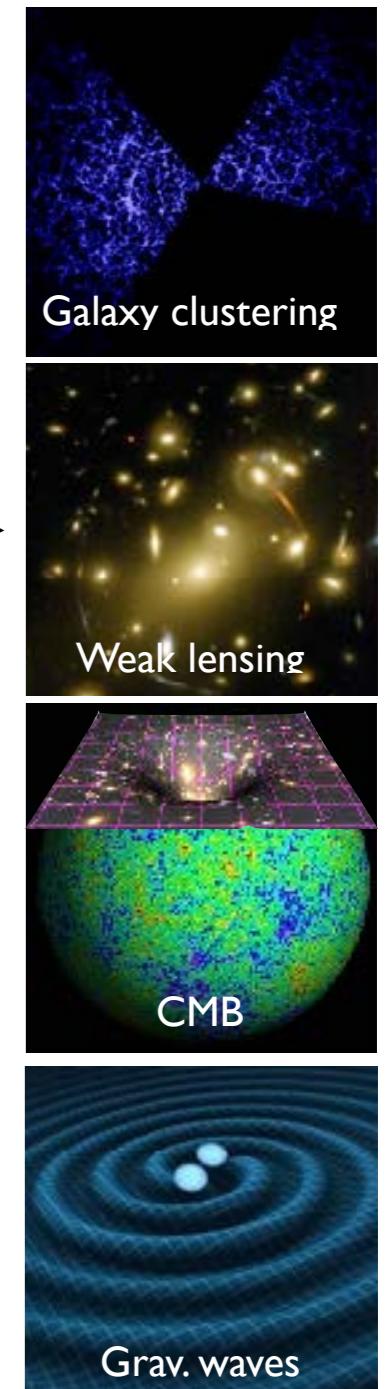
## Models



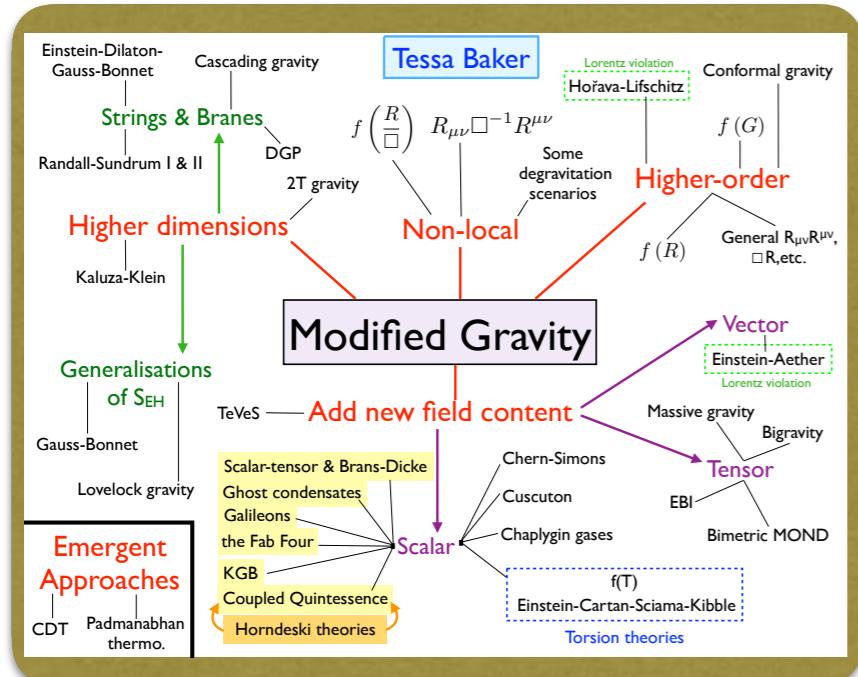
**ETofDE**  
 $\alpha_K(t), \alpha_B(t), \alpha_M(t),$   
 $\alpha_T(t), \alpha_H(t), \dots$

Bridge models and observations in a minimal and systematic way

# Observations



## Models



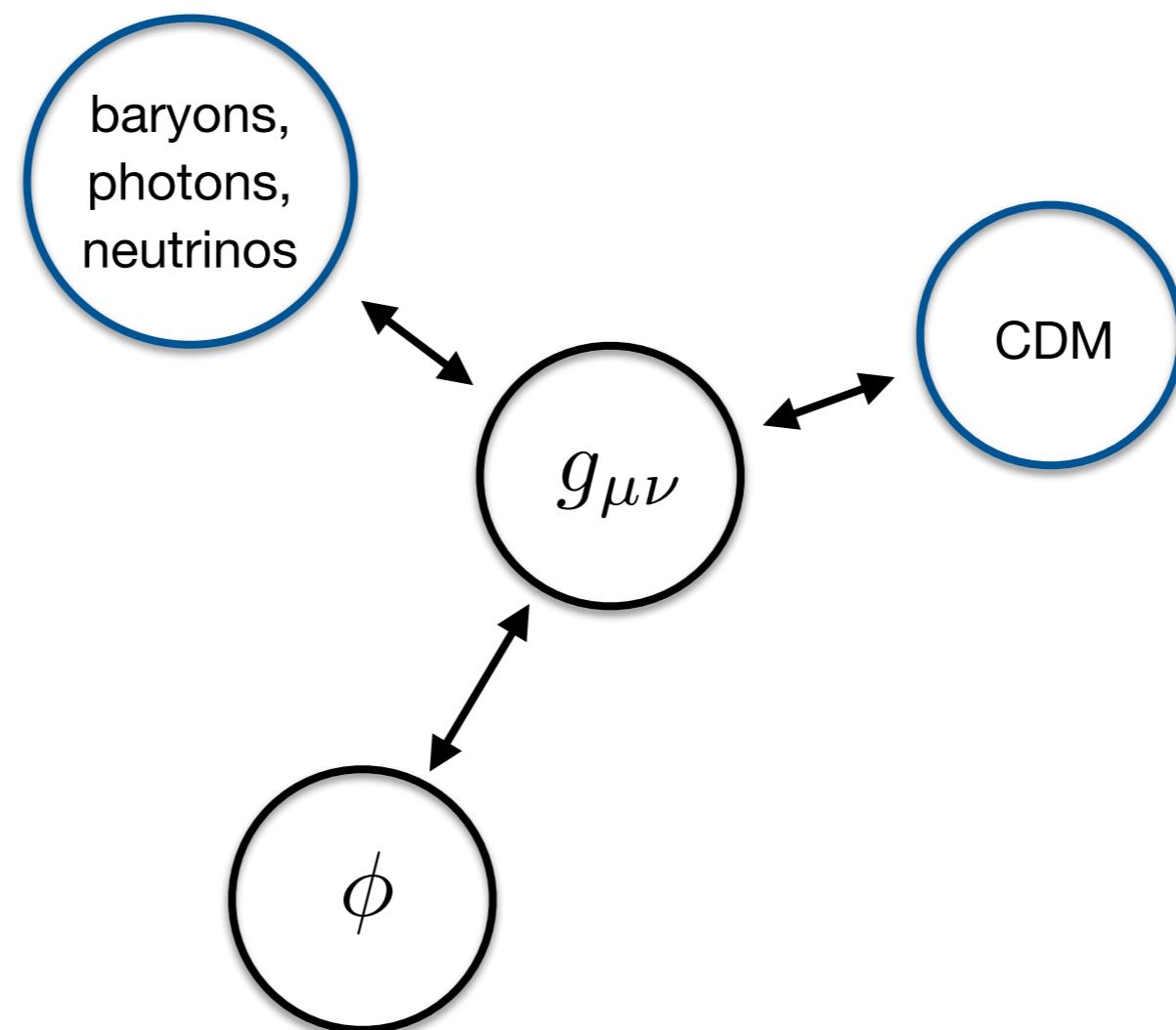
**ETofDE**  
 $\alpha_K(t), \alpha_B(t), \alpha_M(t),$   
 $\alpha_T(t), \alpha_H(t), \dots$

Bridge models and observations in a minimal and systematic way

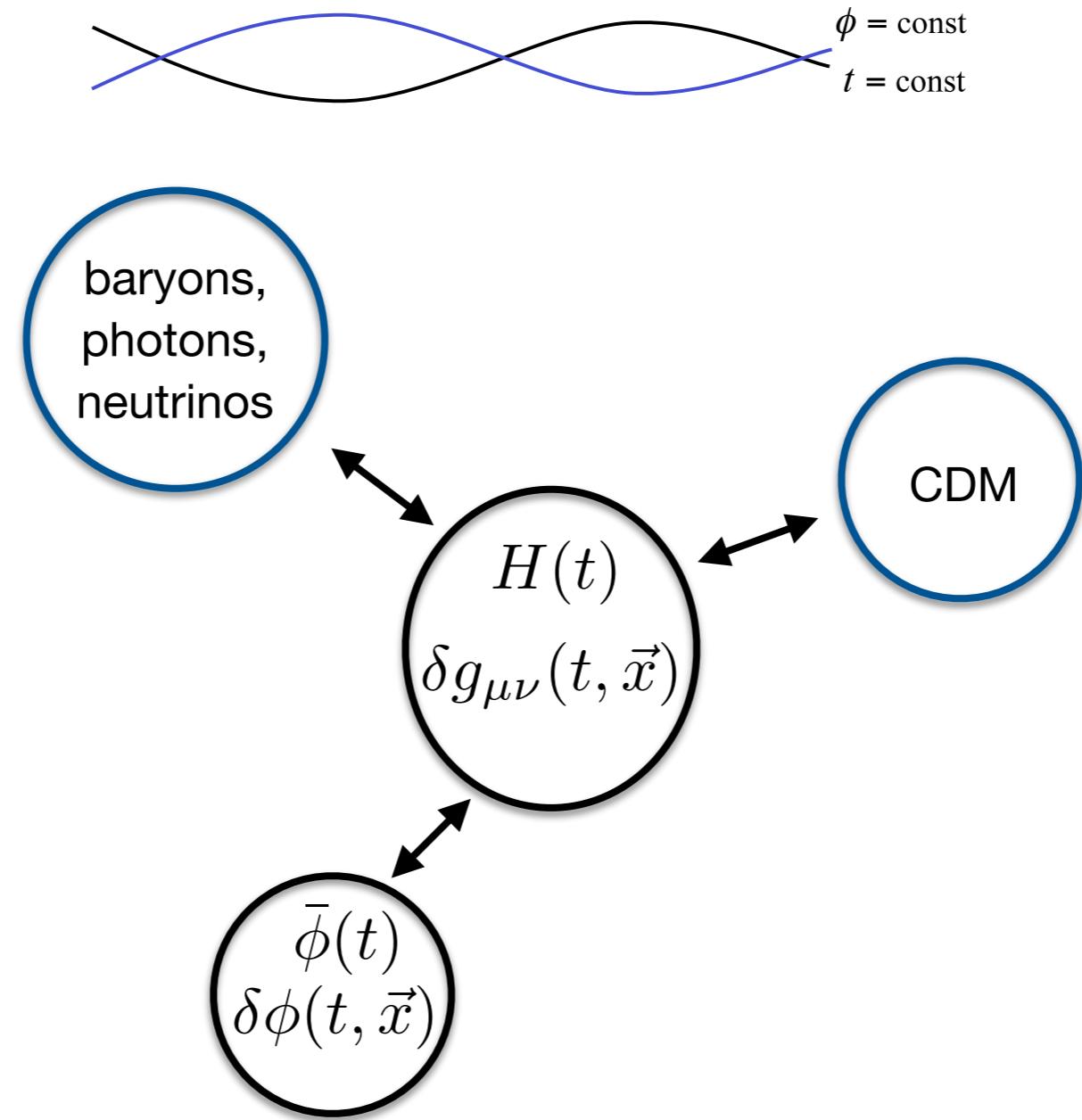
**Theoretically motivated:** locality, causality, unitarity, stability, etc...

**Observationally motivated:** efficiency, now implemented in Einstein-Boltzmann codes

## Jordan frame

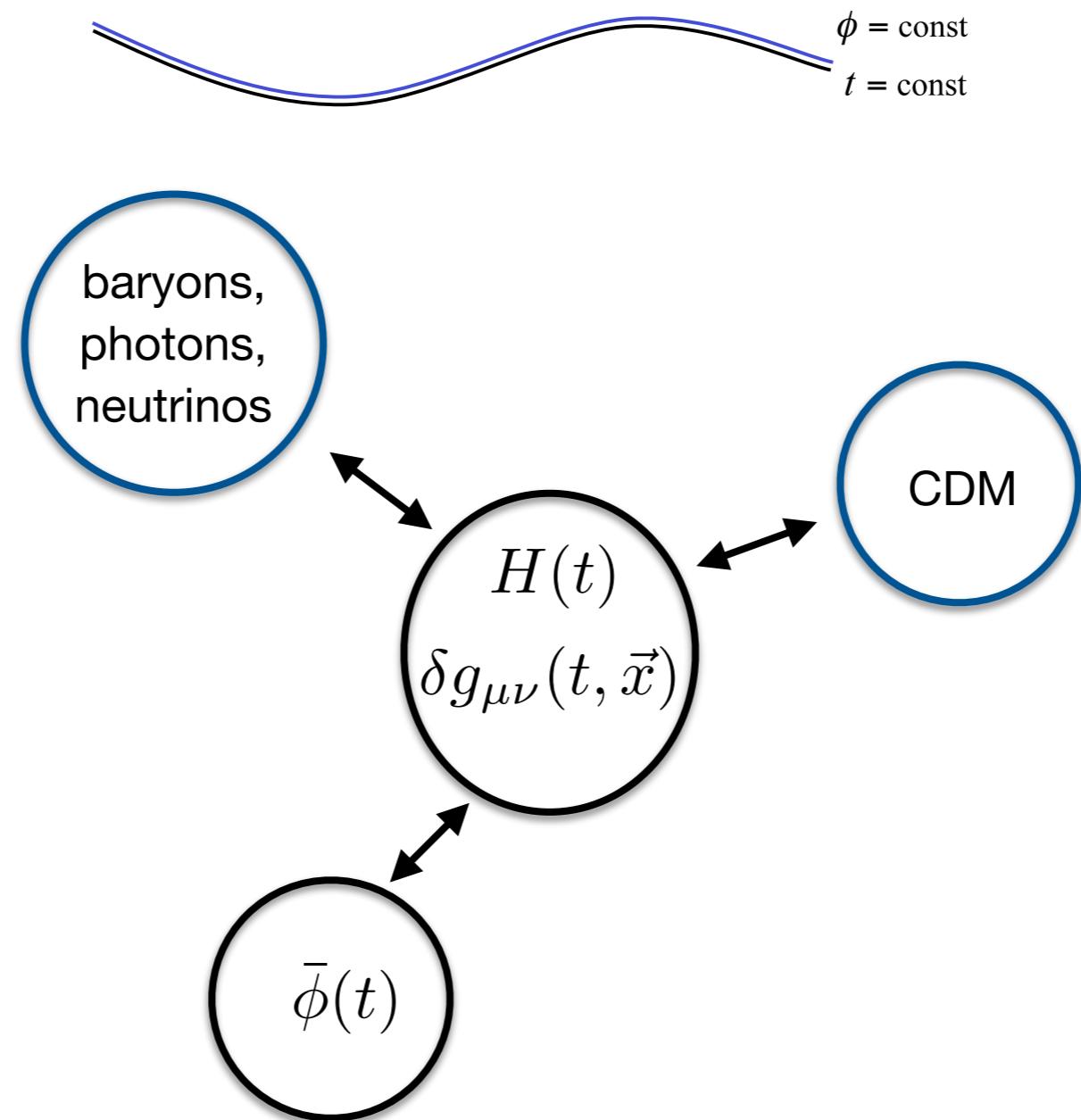


## FLRW background



Time reparametrisation invariance broken,  $\dot{\phi}(t) \neq 0$

**Uniform field slicing**  $\delta\phi(t, \vec{x}) = 0$



Spatial reparametrisation invariance preserved on these hypersurfaces

**Action:** most general function of the metric pert., **preserving spatial-diff** invariance

# EFT Lagrangian

Gubitosi, Piazza, FV 12

Gleyzes, Langlois, Piazza, FV 13

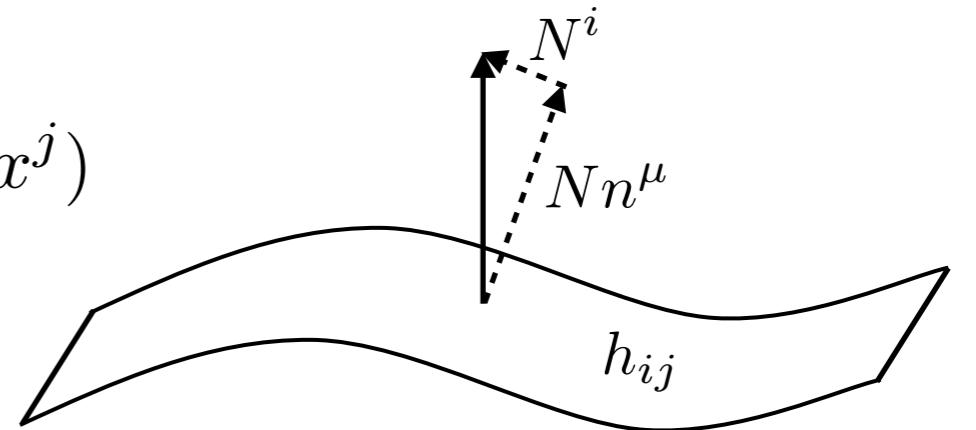
Bloomfield et al. 12, 13

$$S^{(2)} = \int d^4x \sqrt{h} \frac{M^2}{2} \left[ \delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)}(\delta N, \delta K, \dots) \right]$$

ADM decomposition

$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

$$N \sim \dot{\phi}, \quad K_{ij} \sim \dot{h}_{ij}, \quad {}^{(3)}R \sim \partial^2 h$$



# EFT Lagrangian

Gubitosi, Piazza, FV 12

Gleyzes, Langlois, Piazza, FV 13

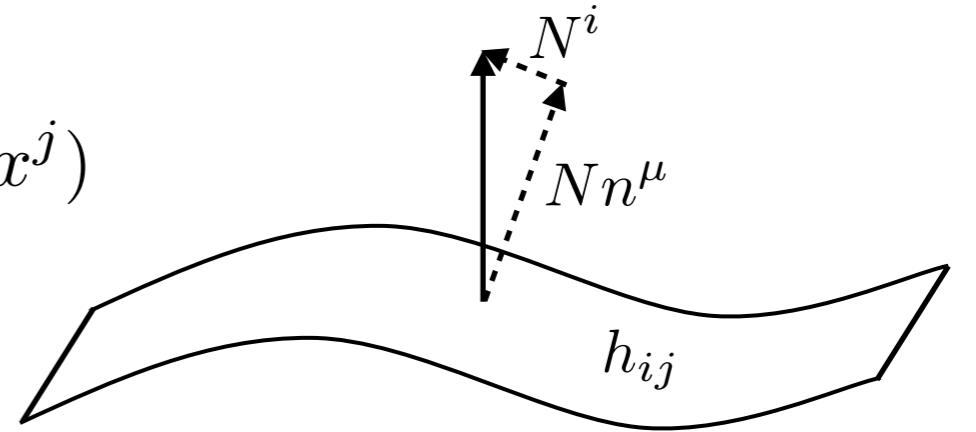
Bloomfield et al. 12, 13

$$S^{(2)} = \int d^4x \sqrt{h} \frac{M^2}{2} \left[ \delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)}(\delta N, \delta K, \dots) \right]$$

ADM decomposition

$$ds^2 = -N^2 dt^2 + h_{ij}(N^i dt + dx^i)(N^j dt + dx^j)$$

$$N \sim \dot{\phi}, \quad K_{ij} \sim \dot{h}_{ij}, \quad {}^{(3)}R \sim \partial^2 h$$



- New operators describe **deviations from GR** ( $\Lambda$ CDM). Ordered in **number of perturbations** and **derivatives**
- **Time-dependent couplings** (functions  $a_i(t)$ ), due to expansion around FLRW background
- Functions  $a_i(t)$  **independent** of background evolution  $H(t) = \dot{a}/a$

We fit to data  $H(t)$  and  $\alpha_i(t)$  (agnostic of their time dependence and parametrization)

# EFT Lagrangian

Gubitosi, Piazza, FV 12

Gleyzes, Langlois, Piazza, FV 13

Bloomfield et al. 12, 13

$$S^{(2)} = \int d^4x \sqrt{h} \frac{M^2}{2} \left[ \delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)}(\delta N, \delta K, \dots) \right]$$

Notation of Bellini, Sawicki '14 for the alphas

	$\alpha_K$	$\alpha_B$	$\alpha_M$	$\alpha_T$
	$\delta N^2$	$\delta N \delta K$	$\frac{dM^2}{d \ln a}$	$\delta_2 {}^{(3)}R$
	kineticity	braiding	conformal coupling	tensor sound speed
quintessence, k-essence	✓			
DGP, kinetic braiding	✓	✓		
Brans-Dicke, f(R)	✓	✓	✓	
Horndeski	✓	✓	✓	✓

**4 functions of time** instead of 4 functions of  $\phi, (\partial\phi)^2$ ; minimal number of parameters

$$N \sim \dot{\phi}, \quad K_{ij} \sim \dot{h}_{ij}, \quad {}^{(3)}R \sim \partial^2 h$$

# EFT Lagrangian

Gubitosi, Piazza, FV 12

Gleyzes, Langlois, Piazza, FV 13

Bloomfield et al. 12, 13

$$S^{(2)} = \int d^4x \sqrt{h} \frac{M^2}{2} \left[ \delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)}(\delta N, \delta K, \dots) \right]$$

Notation of Bellini, Sawicki '14 for the alphas

	$\alpha_K$	$\alpha_B$	$\alpha_M$	$\alpha_T$		
	$\delta N^2$	$\delta N \delta K$	$\frac{dM^2}{d \ln a}$	$\delta_2 {}^{(3)}R$		
	kineticity	braiding	conformal coupling	tensor sound speed		
quintessence, k-essence	✓				No ghosts	Scalar
DGP, kinetic braiding	✓	✓			No gradient inst.	Tensor
Brans-Dicke, f(R)	✓	✓	✓		$\alpha_K + 6\alpha_B^2 > 0$	$M^2 > 0$
Horndeski	✓	✓	✓	✓	$c_s^2(\alpha_i) \geq 0$	$\alpha_T \geq -1$

# EFT Lagrangian

Gubitosi, Piazza, FV 12  
 Gleyzes, Langlois, Piazza, FV 13  
 Bloomfield et al. 12, 13

$$S^{(2)} = \int d^4x \sqrt{h} \frac{M^2}{2} \left[ \delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)}(\delta N, \delta K, \dots) \right]$$

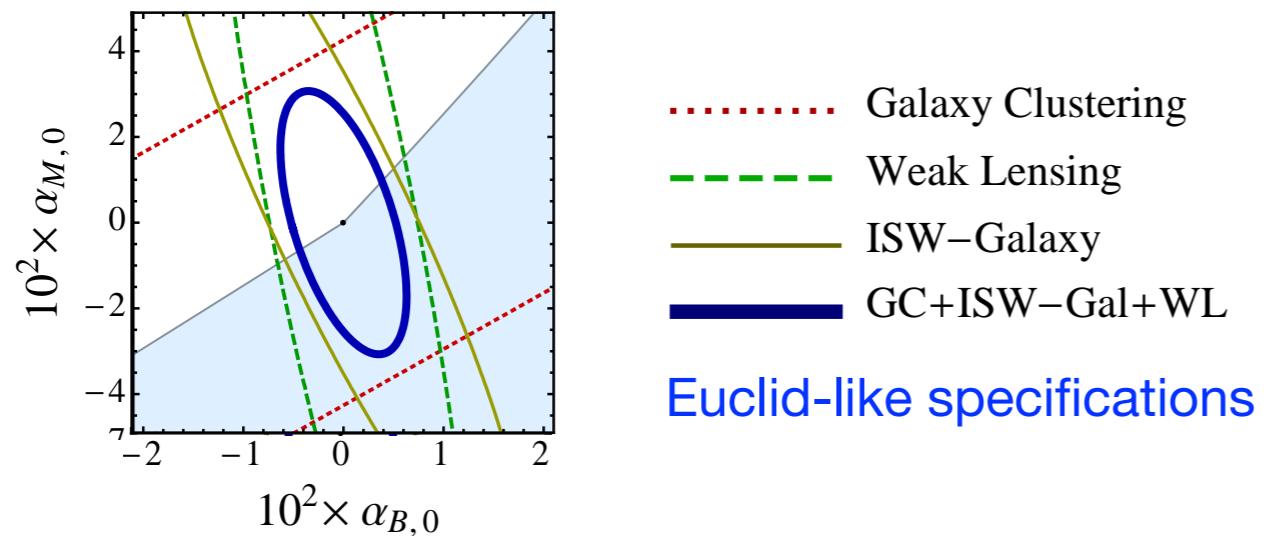
Notation of Bellini, Sawicki '14 for the alphas

	$\alpha_K$	$\alpha_B$	$\alpha_M$	$\alpha_T$
	$\delta N^2$	$\delta N \delta K$	$\frac{dM^2}{d \ln a}$	$\delta_2 {}^{(3)}R$
	kineticity	braiding	conformal coupling	tensor sound speed

**4 functions of time** instead of 4 functions of  $\phi, (\partial\phi)^2$ ; minimal number of parameters

$$N \sim \dot{\phi}, \quad K_{ij} \sim \dot{h}_{ij}, \quad {}^{(3)}R \sim \partial^2 h$$

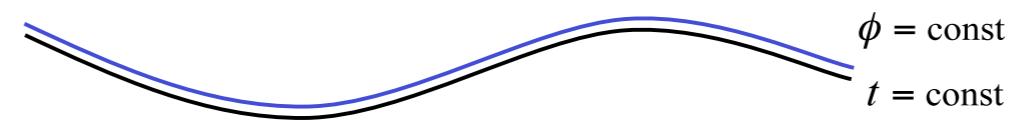
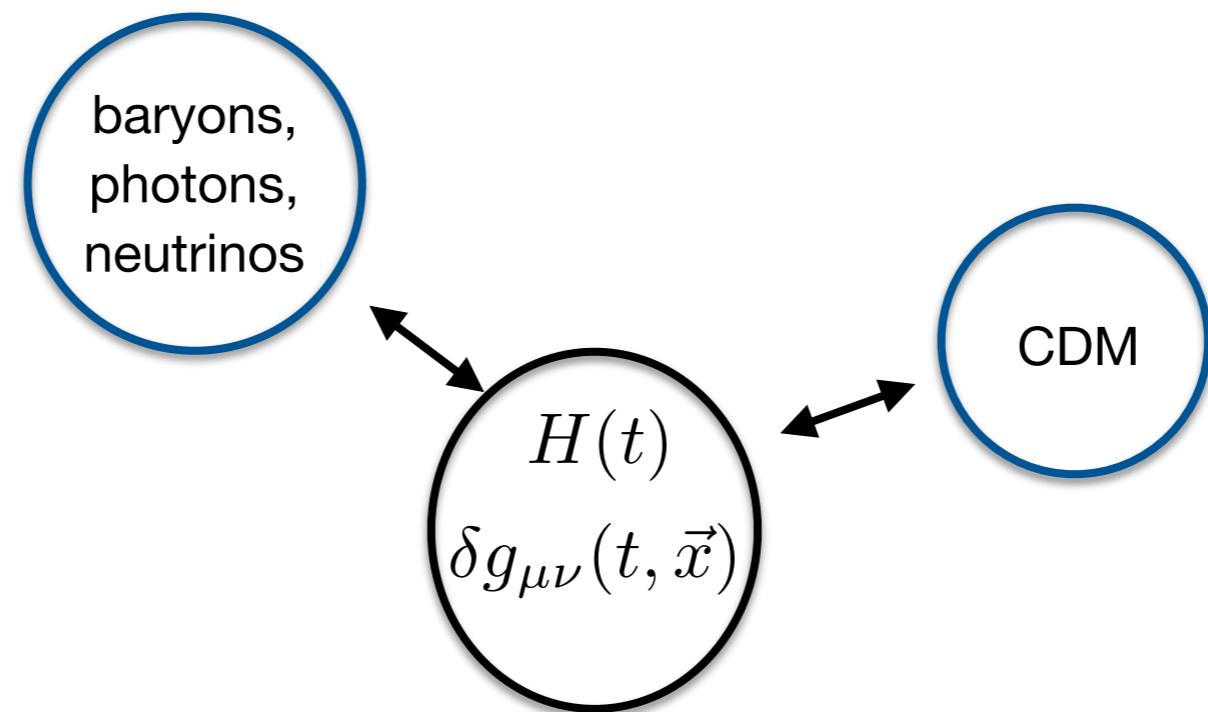
Gleyzes, Langlois, Mancarella, FV '15



	Scalar	Tensor
No ghosts	$\alpha_K + 6\alpha_B^2 > 0$	$M^2 > 0$
No gradient inst.	$c_s^2(\alpha_i) \geq 0$	$\alpha_T \geq -1$

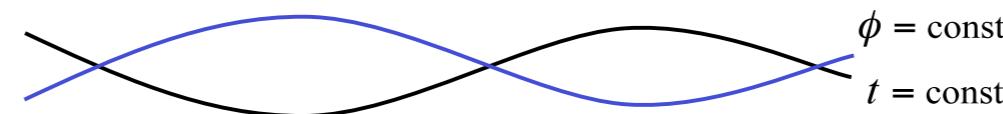
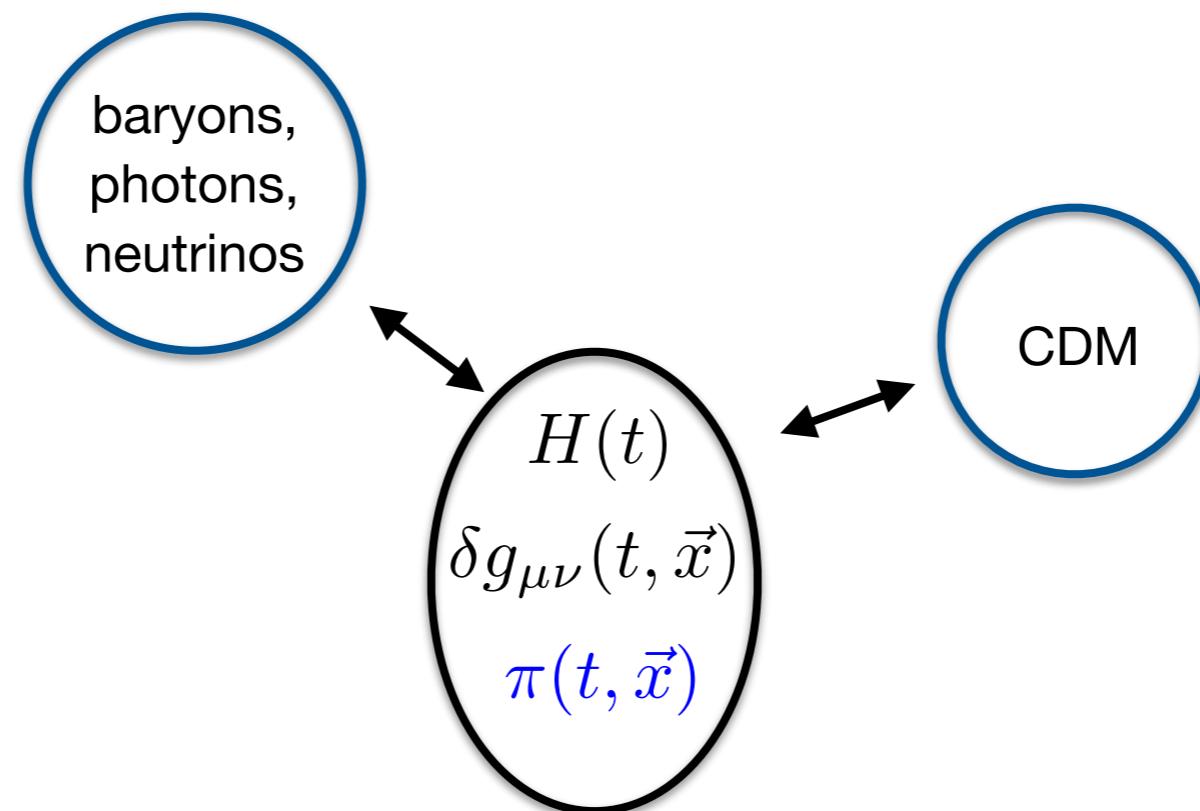
- ♦ Regime of interest for LSS surveys:  $\alpha_i \sim 0.1 \div 0.01$

**Uniform field slicing**  $\delta\phi(t, \vec{x}) = 0$



**Newtonian gauge**  $ds^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)d\vec{x}^2$

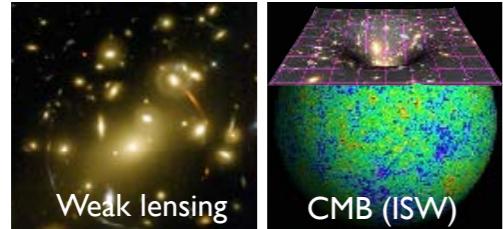
$$t \rightarrow t + \pi(t, \vec{x})$$



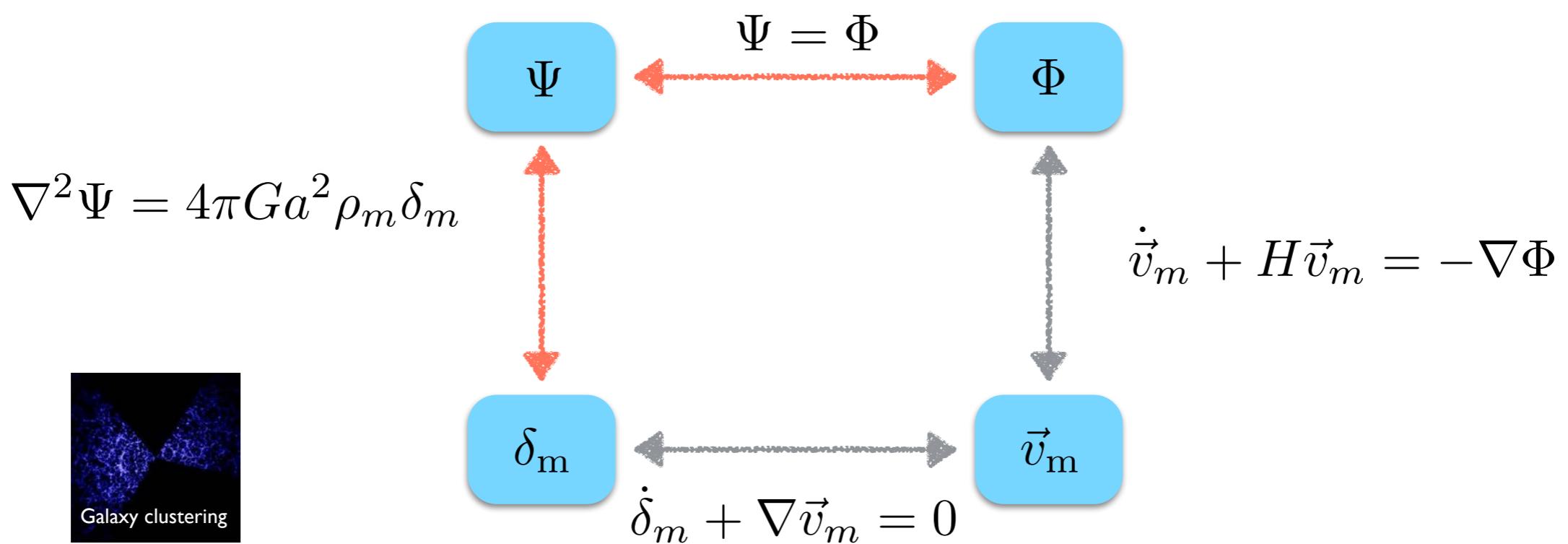
# Phenomenology

$$dt^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)d\vec{x}^2$$

Quasi-static approximations



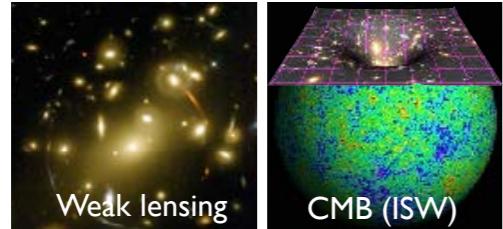
$$\nabla^2(\Psi + \Phi) = 8\pi G a^2 \rho_m \delta_m$$



# Phenomenology

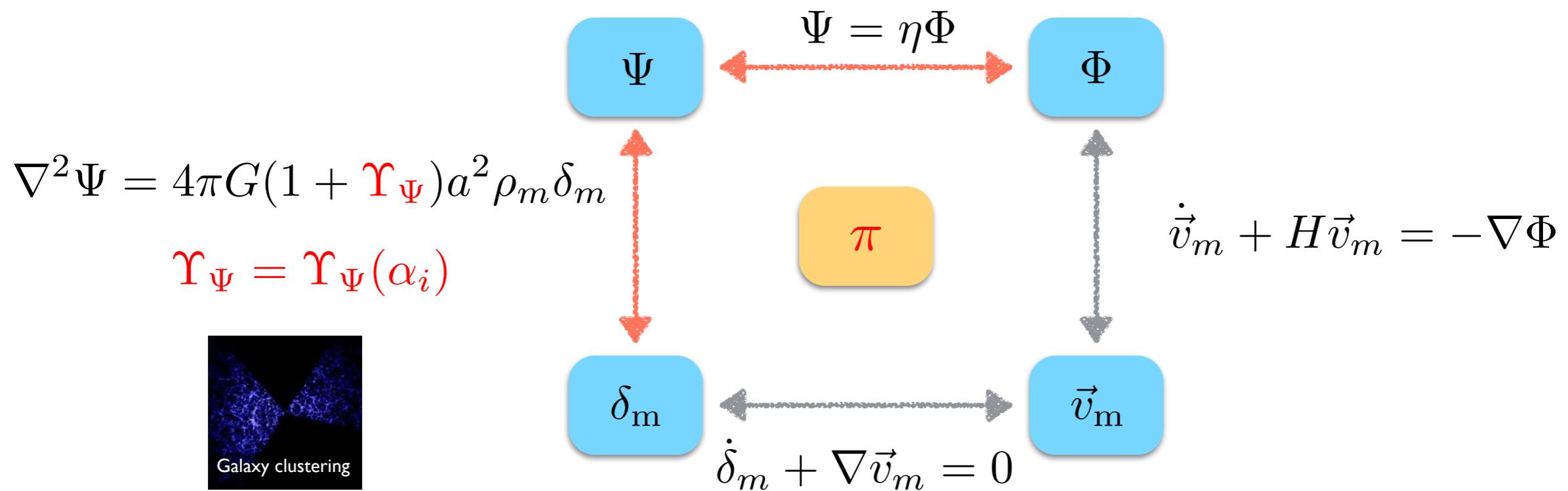
$$dt^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)d\vec{x}^2$$

Quasi-static approximations



$$\nabla^2(\Psi + \Phi) = 8\pi G(1 + \Upsilon_{\text{lens}})a^2\rho_m\delta_m$$

$$\Upsilon_{\text{lens}} = \Upsilon_{\text{lens}}(\alpha_i)$$



# Einstein-Boltzmann solvers

- Full Einstein-Boltzmann solver:

$$\frac{df_I}{d\eta} = C_I[f_I], \quad I = \gamma, \nu, b, \text{CDM}$$

$$\frac{\delta S^{(2)}}{\delta \pi} = 0 \quad \& \quad G_{ij}^{\text{modified}} = 8\pi G \sum_I T_{ij}^{(I)}$$

- EFTCAMB (from CAMB) [\(Hu, Raveri, Frusciante, Silvestri et al.\)](#)
- hi\_class (from CLASS) [\(Zumalacarregui, Bellini, Sawicki, Lesgourgues, Ferreira et al.\)](#)
- COOP (indep. code, Zhiqi Huang) [\(with D'Amico, Huang and Mancarella\)](#)

# Einstein-Boltzmann solvers

- Full Einstein-Boltzmann solver:

$$\frac{df_I}{d\eta} = C_I[f_I], \quad I = \gamma, \nu, b, \text{CDM}$$

$$\frac{\delta S^{(2)}}{\delta \pi} = 0 \quad \& \quad G_{ij}^{\text{modified}} = 8\pi G \sum_I T_{ij}^{(I)}$$

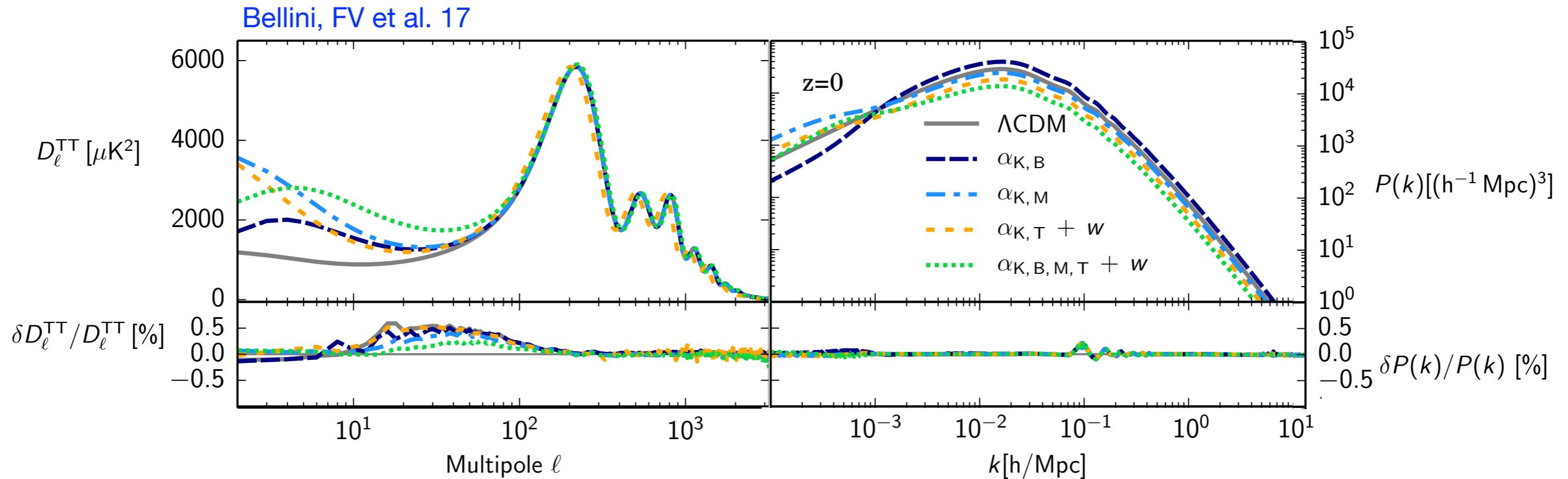
Codes **agree at sub-percent level**, in most cases

- EFTCAMB (from CAMB)

(Hu, Raveri, Frusciante, Silvestri et al.)

- hi\_class (from CLASS)

(Zumalacarregui, Bellini, Sawicki, Lesgourgues, Ferreira et al.)



# Nonlinear EFT of dark energy

$$S = \int d^4x \sqrt{h} \frac{M^2}{2} \left[ \delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)} + \sum_i \alpha_i(t) \mathcal{O}_i^{(3)} \right]$$

Cusin, Lewandowski, FV in prep.

- ◆ Quasi-static regime: only two cubic and one quartic operator

<b>Cubic</b>	$\alpha_{V1} \delta N \delta \mathcal{K}_2$	$\alpha_{V2} \delta \mathcal{K}_3$	$\alpha_{V2} \delta N \delta \mathcal{G}_2$
<b>Quartic</b>	$\alpha_{V3} \delta N \delta \mathcal{K}_3$		

$$\delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_i^j \delta K_j^i , \quad \delta \mathcal{G}_2 \equiv \delta K_i^j R_j^i - \delta K R / 2 ,$$

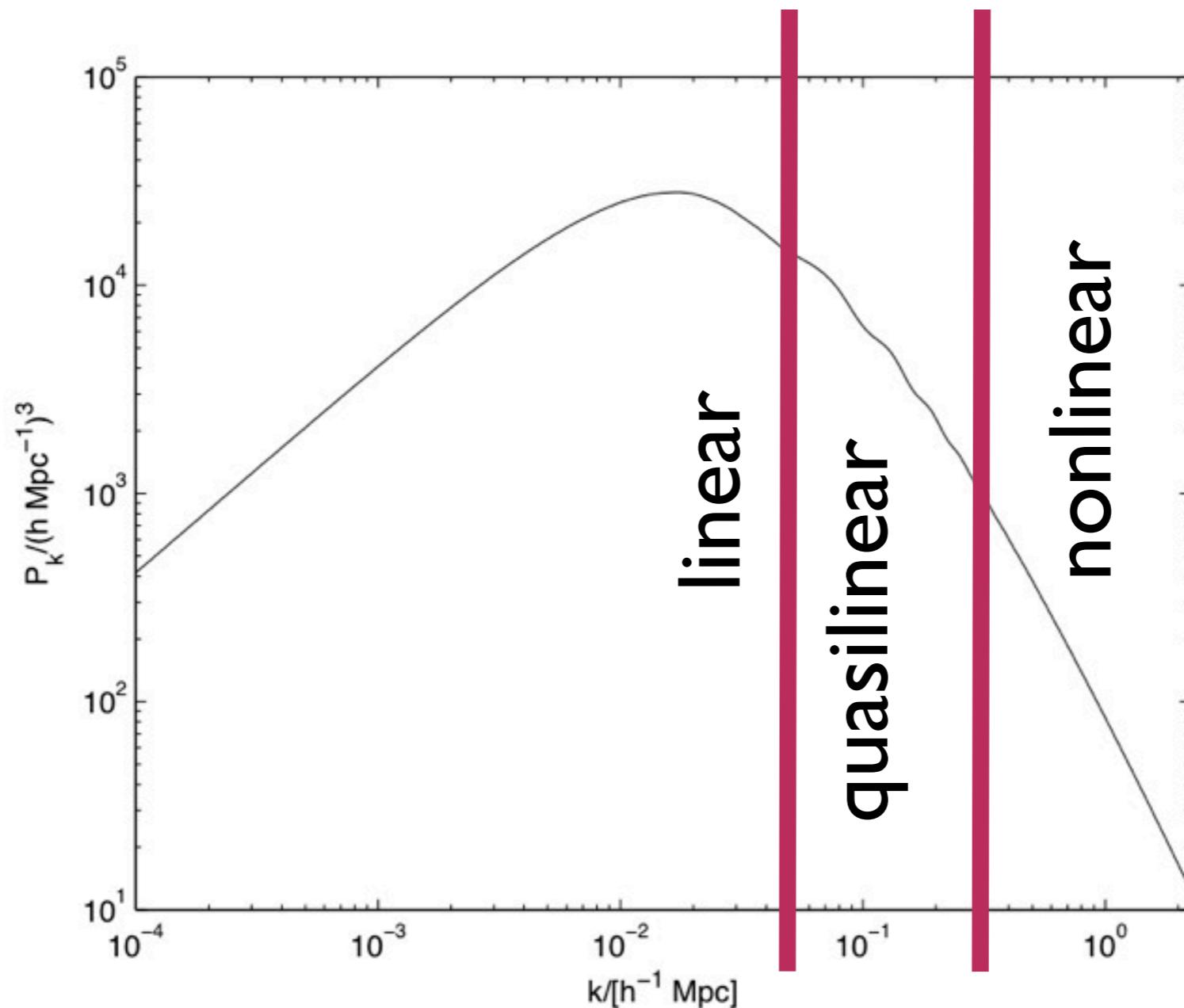
$$\delta \mathcal{K}_3 \equiv \delta K^3 - 3\delta K \delta K_i^j \delta K_j^i + 2\delta K_i^j \delta K_k^i \delta K_j^k$$

- ◆ Nonlinear operators are relevant for Vainshtein screening and structure formation

# Beyond linear regime

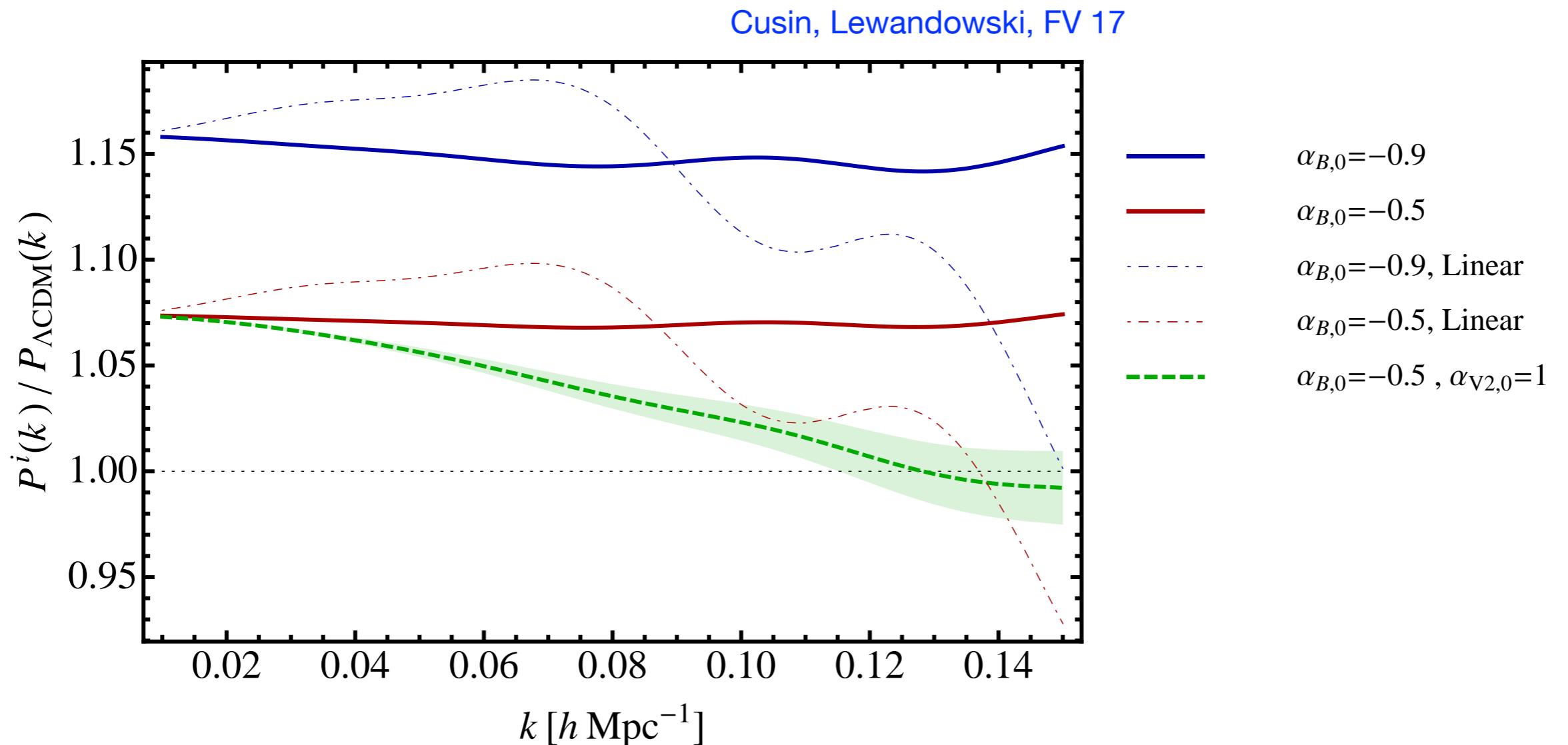
$$\delta(t, \vec{x}) = \rho_m(t, \vec{x}) / \bar{\rho}_m(t) - 1$$

$$\langle \delta(\vec{k}) \delta(\vec{k}') \rangle = (2\pi)^3 \delta_D(\vec{k} + \vec{k}') P(k)$$



# Nonlinear corrections

- ◆ EFT of large-scale structure: perturbative approach accounting for effect of short-scale physics  
[Baumann et al. '10; Senatore et al. '12, etc...](#)
- ◆ Combined with EFT of dark energy: one-loop power spectrum - one new free parameter



# $C_T = 1$ in the EFT of dark energy

Creminelli and FV 17

$$S = \int d^4x \sqrt{h} \frac{M^2}{2} \left[ \delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)} + \sum_i \alpha_i(t) \mathcal{O}_i^{(3)} \right]$$

Quadratic	$\alpha_B \delta N \delta K$	$\alpha_M = \frac{dM^2}{d \ln a}$	$\alpha_T {}^{(3)}R$
Cubic	$\alpha_{V1} \delta N \delta \mathcal{K}_2$	$\alpha_{V2} \delta \mathcal{K}_3$	$\alpha_{V2} \delta N \delta \mathcal{G}_2$
Quartic	$\alpha_{V3} \delta N \delta \mathcal{K}_3$		

$$\delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_i^j \delta K_j^i , \quad \delta \mathcal{G}_2 \equiv \delta K_i^j R_j^i - \delta K R / 2 ,$$

$$\delta \mathcal{K}_3 \equiv \delta K^3 - 3\delta K \delta K_i^j \delta K_j^i + 2\delta K_i^j \delta K_k^i \delta K_j^k$$

# $c_T = 1$ in the EFT of dark energy

Creminelli and FV 17

$$S = \int d^4x \sqrt{h} \frac{M^2}{2} \left[ \delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)} + \sum_i \alpha_i(t) \mathcal{O}_i^{(3)} \right]$$

Quadratic	$\alpha_B \delta N \delta K$	$\alpha_M = \frac{dM^2}{d \ln a}$	<del><math>\alpha_I {}^{(3)}R</math></del>
Cubic	$\alpha_{V1} \delta N \delta \mathcal{K}_2$	$\alpha_{V2} \delta \mathcal{K}_3$	$\alpha_{V2} \delta N \delta \mathcal{G}_2$
Quartic	$\alpha_{V3} \delta N \delta \mathcal{K}_3$		

$$\delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_i^j \delta K_j^i , \quad \delta \mathcal{G}_2 \equiv \delta K_i^j R_j^i - \delta K R / 2 ,$$

$$\delta \mathcal{K}_3 \equiv \delta K^3 - 3\delta K \delta K_i^j \delta K_j^i + 2\delta K_i^j \delta K_k^i \delta K_j^k$$

- ◆ We want  $c_T = 1$  for any background. Small modifications:  $\delta N_{\text{bkgd}} \sim \dot{\phi}$ ,  $\delta K_{\text{bkgd}} \sim H$

# $c_T = 1$ in the EFT of dark energy

Creminelli and FV 17

$$S = \int d^4x \sqrt{h} \frac{M^2}{2} \left[ \delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)} + \sum_i \alpha_i(t) \mathcal{O}_i^{(3)} \right]$$

Quadratic	$\alpha_B \delta N \delta K$	$\alpha_M = \frac{dM^2}{d \ln a}$	<del><math>\alpha_T {}^{(3)}R</math></del>
Cubic	<del><math>\alpha_{V1} \delta N \delta \mathcal{K}_2</math></del>	<del><math>\alpha_{V2} \delta \mathcal{K}_3</math></del>	<del><math>\alpha_{V2} \delta N \delta \mathcal{G}_2</math></del>
Quartic	<del><math>\alpha_{V3} \delta N \delta \mathcal{K}_3</math></del>		

$$\delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_i^j \delta K_j^i , \quad \delta \mathcal{G}_2 \equiv \delta K_i^j R_j^i - \delta K R / 2 ,$$

$$\delta \mathcal{K}_3 \equiv \delta K^3 - 3\delta K \delta K_i^j \delta K_j^i + 2\delta K_i^j \delta K_k^i \delta K_j^k$$

- ♦ We want  $c_T = 1$  for any background. Small modifications:  $\delta N_{\text{bkgd}} \sim \dot{\phi}$ ,  $\delta K_{\text{bkgd}} \sim H$

Dramatic simplification:  $\alpha_T = \alpha_{V1} = \alpha_{V2} = \alpha_{V3} = 0$

# Beyond Horndeski

Not the end of the story: (invertible) metric redefinitions do not change number of d.o.f.

	OLD SCHOOL SCALAR TENSOR	HORNDESKI
OPERATORS	$\alpha_K, \alpha_B, \alpha_M$	$\alpha_K, \alpha_B, \alpha_M, \alpha_T$
COUPLING	$\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu}$	$\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + D(\phi)\partial_\mu\phi\partial_\nu\phi$

# Beyond Horndeski

Not the end of the story: (invertible) metric redefinitions do not change number of d.o.f.

	OLD SCHOOL SCALAR TENSOR	HORNDESKI	BEYOND HORNDENSKI	DHOST
OPERATORS	$\alpha_K, \alpha_B, \alpha_M$	$\alpha_K, \alpha_B, \alpha_M, \alpha_T$	$\alpha_K, \alpha_B, \alpha_M, \alpha_T, \alpha_H$	$\alpha_K, \alpha_B, \alpha_M, \alpha_T, \alpha_H,$ $\beta_1$
COUPLING	$\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu}$	$\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + D(\phi)\partial_\mu\phi\partial_\nu\phi$	$\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + D(\phi, X)\partial_\mu\phi\partial_\nu\phi$	$\tilde{g}_{\mu\nu} = C(\phi, X)g_{\mu\nu} + D(\phi, X)\partial_\mu\phi\partial_\nu\phi$

Higher-order EOM but degenerate theories:

- ◆ Beyond Horndeski (GLPV) theories:

[Gleyzes, Langlois, Piazza, FV 14](#)

$$g_{\mu\nu} \rightarrow C(\phi)g_{\mu\nu} + D(\phi, X)\partial_\mu\phi\partial_\nu\phi$$

Effective parameter:  $\alpha_H(t)$

- ◆ Degenerate higher-order scalar-tensor (DHOST) theories:

[Langlois, Noui 15; Crisostomi et al. 16](#)

$$g_{\mu\nu} \rightarrow C(\phi, X)g_{\mu\nu} + D(\phi, X)\partial_\mu\phi\partial_\nu\phi$$

Effective parameter:  $\beta_1(t)$

# Consequence of $c_T=1$ - Beyond Horndeski

- ◆ Beyond Horndeski (GLPV) theories with  $c_T = 1$ :

$$L_{c_T=1} \equiv G_4(\phi, X) {}^{(4)}R - \frac{4}{X} G_{4,X}(\phi, X) (\phi^\mu \phi^\nu \phi_{\mu\nu} \square \phi - \phi^\mu \phi_{\mu\nu} \phi_\lambda \phi^{\lambda\nu}) \\ + G_2(\phi, X) + G_3(\phi, X) \square \phi$$

- ◆ DHOST theories with  $c_T = 1$ :  $g_{\mu\nu} \rightarrow C(\phi, X) g_{\mu\nu}$

$$L_{c_T=1} = CG_4 {}^{(4)}R - \frac{4CG_{4,X}}{X} \phi^\mu \phi^\nu \phi_{\mu\nu} \square \phi \\ + \left( \frac{4CG_{4,X}}{X} + \frac{6G_4 C_{,X}^2}{C} + 8C_{,X} G_{4,X} \right) \phi^\mu \phi_{\mu\nu} \phi_\lambda \phi^{\lambda\nu} \\ - \frac{8C_{,X} G_{4,X}}{X} (\phi_\mu \phi^{\mu\nu} \phi_\nu)^2 + G_2(\phi, X) + G_3(\phi, X) \square \phi$$

# Consequence of $c_T=1$ - Beyond Horndeski

- ◆ Beyond Horndeski (GLPV) theories with  $c_T = 1$ :

$$L_{c_T=1} \equiv G_4(\phi, X) {}^{(4)}R - \frac{4}{X} G_{4,X}(\phi, X) (\phi^\mu \phi^\nu \phi_{\mu\nu} \square \phi - \phi^\mu \phi_{\mu\nu} \phi_\lambda \phi^{\lambda\nu}) \\ + G_2(\phi, X) + G_3(\phi, X) \square \phi$$

- ◆ DHOST theories with  $c_T = 1$ :  $g_{\mu\nu} \rightarrow C(\phi, X) g_{\mu\nu}$

$$L_{c_T=1} = CG_4 {}^{(4)}R - \frac{4CG_{4,X}}{X} \phi^\mu \phi^\nu \phi_{\mu\nu} \square \phi \\ + \left( \frac{4CG_{4,X}}{X} + \frac{6G_4 C_{,X}^2}{C} + 8C_{,X} G_{4,X} \right) \phi^\mu \phi_{\mu\nu} \phi_\lambda \phi^{\lambda\nu} \\ - \frac{8C_{,X} G_{4,X}}{X} (\phi_\mu \phi^{\mu\nu} \phi_\nu)^2 + G_2(\phi, X) + G_3(\phi, X) \square \phi$$

In terms of EFT parameters

$$S = \int d^4x \sqrt{h} \frac{M^2}{2} \left[ -\delta \mathcal{K}_2 + {}^{(3)}R + 4\alpha_B H \delta N \delta K \right. \\ \left. + (1 + \alpha_H) \delta N {}^{(3)}R + 4\beta_1 \delta N \dot{\delta K} + \beta_2 \delta N^2 + \beta_3 (\partial_i \delta N)^2 - \alpha_H \delta N \delta \mathcal{K}_2 \right]$$

# Vainshtein screening for stars and pulsars

Dima, FV 17; Crisostomi, Koyama 17; Langlois, Noui 17

In beyond Horndeski models, Vainshtein screening partially broken inside and around compact objects. Constraints on beyond-Horndeski character:  $\alpha_H$  ,  $\beta_1$

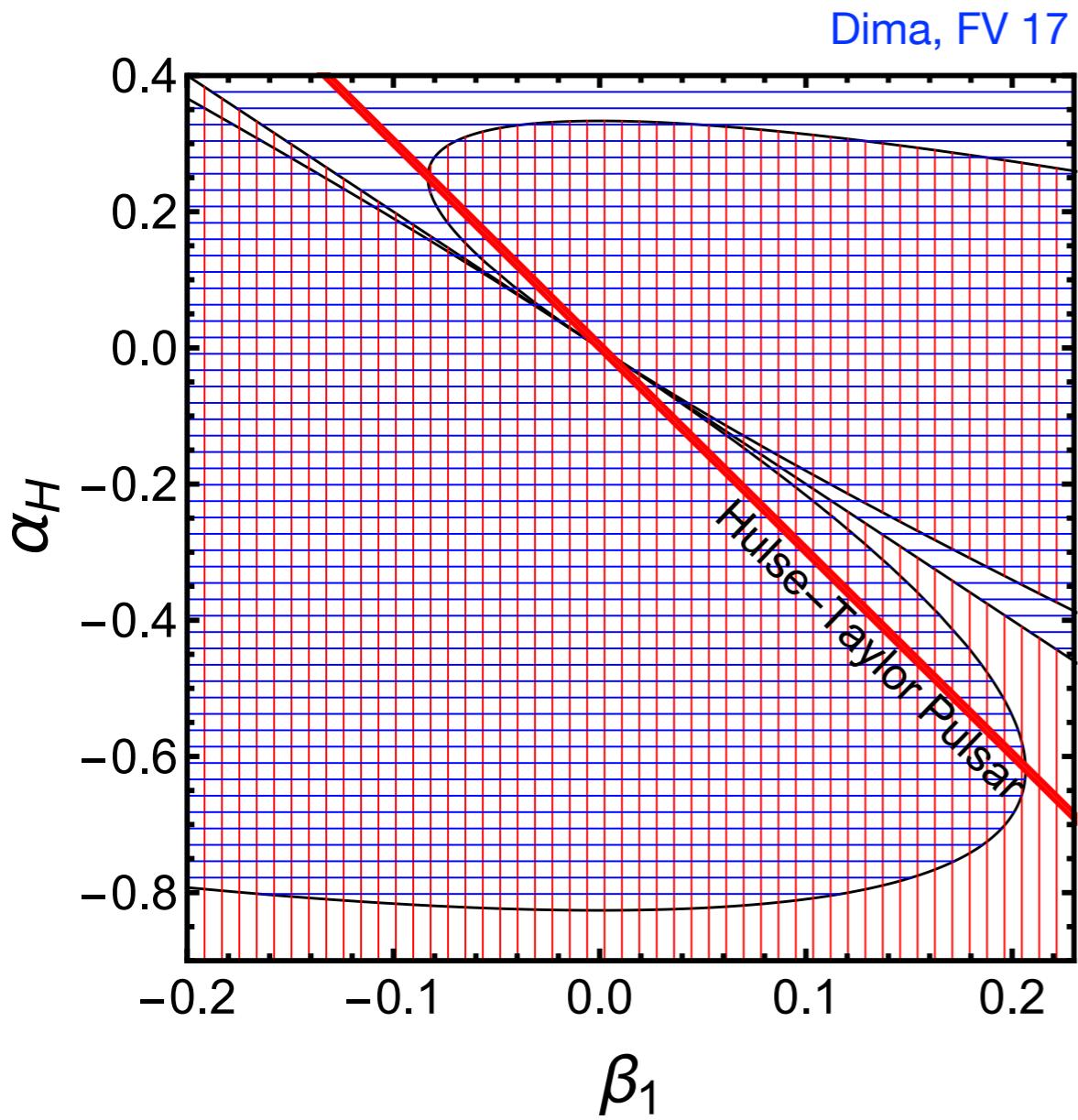
$$\frac{d\Phi}{dr} = G_N \left[ \frac{M(r)}{r^2} + \gamma(\alpha_H, \beta_1) M''(r) \right]$$

$$G_N = (1 - \alpha_H - 3\beta_1)^{-1} G_{\text{gw}}$$

# Vainshtein screening for stars and pulsars

Dima, FV 17; Crisostomi, Koyama 17; Langlois, Noui 17

In beyond Horndeski models, Vainshtein screening partially broken inside and around compact objects. Constraints on beyond-Horndeski character:  $\alpha_H$ ,  $\beta_1$



$$\frac{d\Phi}{dr} = G_N \left[ \frac{M(r)}{r^2} + \gamma(\alpha_H, \beta_1) M''(r) \right]$$

$$G_N = (1 - \alpha_H - 3\beta_1)^{-1} G_{\text{gw}}$$

- ◆ Hydrostatic equilibrium in stars (upper bound on strength of gravity) Saito et al. 15
- ◆ Red dwarf minimal mass (lower bound on strength of gravity) Sakstein 15
- ◆ Hulse-Taylor pulsar Jimenez, Piazza, Velten 15

$$\dot{P} = \frac{G_{\text{gw}}}{G_N} \dot{P}_{\alpha_H=0, \beta_1=0}$$

# Conclusion

- ◆ Testing GR: one of the main targets of future LSS surveys. EFT approach, useful on cosmological, linear and mildly nonlinear, scales.
- ◆ Measurement of speed of GW: dramatic cut in the available models: Higher-derivatives in Horndeski theories are ruled out. Higher derivatives survive only in beyond Horndeski theories.
- ◆ Beyond Horndeski theories are constrained by stellar physics and H-T pulsar
- ◆ GW observations competitive/complementary to what future LSS survey will do

