Quantum Geometry in Cosmology

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1. Motivation
   - Quantum Spacetime and Quantum Gravity
   - Cosmology

2. Fuzzy Singularity

3. Methods for Investigating Possible Signatures in CMB
Motivation
Gravitational Stability Against Localization of Events

Quantum gravity $\Rightarrow$ spacetime uncertainty relations and coarse graining as effective ‘semiclassical’ effect $\rightarrow$ phenomenological approach

**Principle of gravitational stability against localization of events**

Following Heisenberg length measurement to precision $\Delta x$ induces momentum uncertainty $\Delta p$ that allows energy $E \sim (\Delta x)^{-1}$ to localize in region $\Delta x$. Large energy density creates event horizon $r \sim E \sim l_{pl}^2(\Delta x)^{-1}$. The horizon should not hide the length measurement, hence $\Delta x \geq l_{pl}$.

- Refinement: $\Delta t \sum_i \Delta x^i \geq l_{pl}^2$ and $\sum_{i \neq j} \Delta x^i \Delta x^j \geq l_{pl}^2$ (Doplicher, Fredenhagen, Roberts ’95). Valid in Quantum Minkowski and in similar fashion in curved space.
- Gravity + QM $\Rightarrow$ quantization of spacetime $\rightarrow$ noncommutative geometry (effective theory)
- In nonassociative models: Also volume quantization
- NC and NA emerge naturally in quantization: NC $\leftrightarrow$ non-zero magnetic flux; NA $\leftrightarrow$ non-zero magnetic charge. (Jackiw ’04)
Quantum Spacetime – Quantization

Apply quantization procedures to spacetime manifold itself

Quantization

- Quantization map $\rho : A_{CM} \leftrightarrow A_{QM}$ between observable algebras such that classical states $\omega : A_{CM} \rightarrow \mathbb{C}$ (positive, linear functionals) are classical limits of QM states $\tilde{\omega} : A_{QM} \rightarrow \mathbb{C}$.

- Quantization: commutative $\rightarrow$ noncommutative

- Mainly use HAMILTONIAN approach: phase space $\mathcal{P}$, observables $(C(\mathcal{P}), \{., .\}_{PB})$, states $\omega : f \mapsto \int f d\mu(\omega)$, pure states $\delta_x$. In general $A_{CM}$ Poisson-$\ast$-algebra.
  - Canonical: $A_{QM} \cong \mathcal{B}(\mathcal{S})$, $\rho$ defines ordering prescription, e.g. standard or Weyl ordering. Replace PB by commutator $\leftrightarrow$ non-trivial Lie integration of Poisson to Quantomorphisms.
  - Deformation: $A_{QM} \cong (C(\mathcal{P})[[\lambda]], \ast)$. $\rho$ defines $\ast$-product (connected to ordering and symmetries).
  - Geometric: Here Schrödinger picture is used and $\mathcal{S}$ of states is constructed from symplectic $(\mathcal{P}, \omega)$.
  - Constraints: Symmetry vs reduction
  - Points in phase space no longer states in QM! $\Rightarrow$ strict locality lost

- Quantum Spacetime/Noncommutative Geometry: Find proper quantization map for spacetime, e.g. for de Sitter, that reduces to classical spacetime in some classical limit. Then: NC gauge theory, NC gravity, etc.

- For example NC gravity:
  1. Deformed diffeomorphisms
  2. Gravity as NC gauge theory
Quantum Spacetime – Examples of Quantum Spacetimes and Quantization Strategies

Examples/Quantization Strategies

• Often: Impose classical symmetries and extend spacetime to larger spacetime including symmetry generators (e.g. Buric, Madore for dS or Steinacker for FLRW.)

• **Matrix Models:** Often IKKT (reduce SYM to 0D) or BFSS (reduce SYM to 1D). Spacetime $\hat{=} \text{spectrum of matrices}$. Can construct coherent states in this approach. Application of IKKT to cosmology due to H. Steinacker ($\rightarrow$ singularity $\hat{=} \text{signature change in hyperbolic model [last colloquium at Jacobs]}$)

• **Doplicher, Fredenhagen, Roberts ’95 (DFR):**

\[
[\hat{q}^\mu, \hat{q}^\nu] = i\hat{Q}^{\mu\nu}
\]
\[
[\hat{Q}^{\mu\nu}, \hat{q}^\rho] = 0
\]
\[
\hat{Q}_{\mu\nu} \hat{Q}^{\mu\nu} = 2(\hat{m}^2 - \hat{e}^2) = 0
\]
\[
(\hat{Q}_{\mu\nu}(\ast \hat{Q})^{\mu\nu})^2 = 4([\hat{e} \cdot \hat{m}]_+)^2 = 16 \cdot \hat{1}
\]

**Important properties:** Unitarity despite space-time NC and Lorentz invariance despite coarse graining

• **BTZ black hole quantization:** (Dolan, Gupta, Stern ’09) Use geometric identity BTZ $= AdS^3/\sim$, then contract Poisson brackets compatible with symmetry $\Rightarrow \hat{t}$-spectrum $n\tau - a\tau/2\pi$ where $\alpha = a\tau/(2\pi)$ labels irreps.
Problems of Standard (non-inflationary) Cosmology

- **Horizon problem:** See next slide

- **Flatness problem:** \( \Omega(t) := \sum_{i=\text{mat, rad, } } \Omega_i(t) \Rightarrow |\Omega - 1| (t) = \frac{|k|}{(aH)^2(t)} \). Hubble horizon \( aH \) is shrinking \( \Rightarrow |\Omega - 1| = 0 \) unstable point. Again fine-tuning problem: \( |\Omega - 1| (t_{pl}) < 10^{-60} \).

- **Monopole problem:** (Preskill ’79) GUTs produce magnetic monopoles, cosmic evolution too slow to reduce density below current upper bounds.

- **Structure formation problem:** Non-inflationary evolution does not enhance primordial quantum fluctuations enough to account for fluctuations in CMB and today’s structure in universe.

All fine-tuning problems
Horizon Problem – Causal Structure

Observer today

Backward light cone

Singularity

\( z = \infty \)

\( z \approx z_{\text{rec}} \approx 1100 \)

\[
d s^2 = a^2(\eta)(-d\eta^2 + \gamma) = -dt^2 + a^2(t)\gamma, \quad z + 1 = a_0/a
\]

\[
\frac{l_{\text{univ}}}{l_{\text{LSS}}} = \frac{\int_0^{a_0} \frac{da'}{H(a')(a')^2}}{\int_0^{a_{\text{LSS}}} \frac{da'}{H(a')(a')^2}} = \frac{\int_0^{a_{\text{LSS}}} \frac{da'}{H(a')(a')^2}}{\int_0^{a_{\text{LSS}}} \frac{da'}{H(a')(a')^2}}. \quad \text{for MD } a \sim t^{2/3}, \ H \sim a^{-3/2}, \ \text{for RD } a \sim t^{1/2}, \ H \sim a^{-2},
\]

hence largest contribution from MD \( \Rightarrow \left( \frac{l_{\text{univ}}}{l_{\text{LSS}}} \right)^2 \approx \left( \frac{\sqrt{a_0}}{\sqrt{a_{\text{LSS}}}} \right)^2 = 1 + z_{\text{LSS}} \approx 1100. \) Fine-tuning problem.
**Problem:** Inflation only accurate down to $t = t_{\text{pl}} \Rightarrow$ horizon problem strikes back at some $t \gg t_0$
**Time and Length Scales**

**Time**

- In general evolution time $\neq$ coordinate time. Consider e.g. Hamiltonian constraint in Canonical QG $\hat{H}\lvert\Psi\rangle = 0$ (Wheeler-deWitt) $\rightarrow$ no explicit time evolution, but statics? No! Relative evolution.

- A priori: entropic time $\neq$ coordinate time $\neq$ relative time

**Length Scales**

- Consider standard cosmology. QG effects become important when $T \approx T_{pl}$. Then fundamental length scale $l_{pl}$ defined with respect to physical distance $\Delta s \sim l_{pl}$, but often $\Delta x \sim l_{pl}$. But $\Delta x \sim a^{-1}\Delta s$, hence $\Delta x \sim a l_{pl}$?

- Extra dimensions: $l_{pl}^{(4)} = \frac{(l_{pl}^{(D)})^{D/2-1}}{\sqrt{V}}$ (Arkani-Hamed, Dimopoulos, Dvali '98) $\rightarrow$ Planck length might be smaller than it appears.

- Planck’s constant might really be operator (Heckmann, Verlinde '15) $[\hat{x}, \hat{p}] \sim \hat{h}$. Define Planck length $\hat{l}_{pl}^2 = \hat{h}G_N/c^3$ such that length scales $\in$ spec($\hat{l}$)?

- Planck mass might be even variable in space and time: variable gravity (Wetterich '14) $M_{pl}^{eff} \sim \chi(t, \vec{x})$ cosmon.
Fuzzy Singularity
Horizon Problem – Initial Time Uncertainty

\[ \Delta x(t<<1) = \frac{\Delta t}{|t'(x)|} \bigg|_{t=0} = \infty \]

In radiation domination: Small time uncertainty \( \Rightarrow \) infinite space uncertainty
Fuzzy Singularity I – Heuristic Idea

Imposing small time uncertainty at early times leads to fuzzy light cones

“dusty” singularity and “stacked” multiverses. For horizon problem no need of inflation. Problem: How to “average”? 
Space-time commutation relations

\[
[\hat{t}, \hat{x}^j] = i\lambda \hat{\alpha}^j(t, \vec{r}) \\
[\hat{\alpha}^j, \hat{x}^k] = [\hat{\alpha}^j, \hat{t}] = 0
\]

with \( \hat{\alpha} \) vector-valued Hermitian operator, that transforms covariantly and is central in the algebra.

Uncertainty: \( \Delta x^i \Delta t \geq \frac{1}{2} \alpha^j(t, \vec{r}) \) with \( \alpha^j := |\langle \hat{\alpha}^j \rangle| \). Lightlike curve satisfies \( |\dot{\gamma}_s|(t) = \frac{1}{|a(t)|} \) in comoving coordinates. Impose \( \Delta t \) at some early time → geodesic inherits uncertainty

\[
\Delta \gamma_s(t, \vec{r}) = \frac{\Delta t}{a(t)} \geq \frac{1}{2} \frac{\alpha^j(t, \vec{r})}{|a|} \frac{1}{\Delta x}. \quad \text{If } \Delta x = \Delta \gamma_s \Rightarrow
\]

\[\Delta x^i \geq \sqrt{\frac{\lambda \alpha^j}{2|a|}} \quad \alpha^j \text{ slowly varying} \quad \infty(t \to 0)\]

\textbf{Note:} Doplicher, Morsella, Pinamonti '13: Related idea. Using DFR-model, showed that 'localization region' diverges for \( t \to 0 \).
How to measure $\Delta t$?

**Deferred Measurement Principle**

In a quantum circuit measurements and operations may be interchanged. $\Rightarrow$ Measurement may be shifted to the end.

- **MEASUREMENT:** Positive operator valued measure (POVM): $\{\hat{E}_i\}$ self-adjoint operators with $\sum_i \hat{E}_i = \hat{1}$. Writing $\hat{\rho} = \sum_i \hat{E}_i\hat{\rho} \rightarrow$ measurement $\hat{\rho} \mapsto \hat{E}_i\hat{\rho}$ with probability $P_i = \text{Tr}(\hat{E}_i\hat{\rho})$.

- **OPERATIONS/QUANTUM CHANNEL:** Completely positive maps between density matrices. In canonical Kraus form: $\hat{\rho} \mapsto \hat{\rho}' = \sum_{i=1}^{r} \hat{A}_i\hat{\rho}\hat{A}_i^\dagger$ with $\sum_{i=1}^{r} \hat{A}_i^\dagger\hat{A}_i = \hat{1}$ and $\text{Tr}(\hat{A}_i^\dagger\hat{A}_j) = d_i\delta_{ij}$, $\sum_{i=1}^{r} d_i = \text{dim}(\mathcal{H})$ with $r$ the Kraus rank.

- Most general temporal evolution given by Lindblad equation (like above plus two additional terms)

$\Rightarrow$ Strategy: Follow photon with energy uncertainty (induced by time uncertainty), then initial time uncertainty should be detectable much later (e.g. on LSS).
Methods for Investigating Possible Signatures in CMB
• Relic black body radiation from LSS at about $z \approx 1100$ shortly after recombination
• Small anisotropies depict energy density fluctuations from primordial quantum fluctuations
• Observation on celestial sphere: \[ \frac{\delta T}{T_0} = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm} \] with $T_0 \approx 2.7$ K and $\frac{\delta T}{T_0} \sim 10^{-5} - 10^{-4}$.
• Question: Is $\delta T$ Gaussian and isotropic random field? Isotropy: (weak/statistical) cosmological principle. Gaussianity: e.g. inflation.

(1) What are QS signatures in CMB? (2) Are they present in the data?
Multipole vectors

Fixed multipole: \( \{ a_{lm} \} \overset{1:1}{\leftrightarrow} \{ \varphi(l,i) \in \mathbb{R}P^2 \} \) depicted as \( l \) unit vectors in one hemisphere. Isotropic and Gaussian temperature fluctuations \( \Rightarrow \) MPVs feel repulsion. not uniformly distributed.

(Pinkwart, Schwarz; in prep)

\[
S(l) = \frac{1}{l} \sum_{i=1}^{l} |\varphi(l,i) \cdot \vec{D}|, \; \vec{D} \text{ cosmic dipole. (Anti-)correlation on large angular scales } \rightarrow \text{ nature of dipole not fully understood or other effects?}
\]
Strategy: pure states yield vanishing von Neumann entropy $\rightarrow$ construct mixed density matrix such that entropy is rotationally invariant.

\[ S_{\text{ang}}(l) = -\text{Tr}(\rho_{\text{ang}}(l) \ln(\rho_{\text{ang}}(l))) \] with $\rho_{\text{ang}}(l) = \frac{1}{l(l+1)} \sum_{i=1}^{3} L_i \rho(l) L_i$, where $\rho(l) = |\Psi_l\rangle\langle\Psi_l|$ and $|\Psi_l\rangle = \sum_{m=-l}^{l} \tilde{a}_{lm} |l, m\rangle$, $\tilde{a}_{lm} = \frac{a_{lm}}{\sum_{n=-l}^{l} a_{ln}}$.

Minimal for Bloch coherent states (?) ; maximal for maximally mixed $\rho_{\text{ang}}$. 

(Pinkwart, Schupp; in prep)
Conclusion

— Phenomenological description of QG $\rightarrow$ Quantum geometry
— Quantizing spacetime $\Rightarrow$ Functions on manifold $\rightarrow$ Noncommutative algebra
— Small initial time uncertainty causally connects all initial points
— Possible tools for investigating signatures in CMB: MPVs and pseudo-entropies