Quantum Geometry in Cosmology

Marvin Pinkwart

19. January 2018 – 10:30 – RTG Models of Gravity Workshop Jacobs University Bremen

Jacobs University Bremen

RESEARCH TRAINING GROUP
Models of Gravity

Table of contents

Motivation

Gravitational Stability Against Localization of Events

Quantum gravity $\Rightarrow$ spacetime uncertainty relations and coarse graining as effective 'semiclassical' effect $\longrightarrow$ phenomenological approach

Principle of gravitational stability against localization of events

Following Heisenberg length measurement to precision Δx induces momentum uncertainty Δp that allows energy $E \sim (\Delta x)^{-1}$ to localize in region Δx. Large energy density creates event horizon $r \sim E \sim l_{pl}^2 (\Delta x)^{-1}$. The horizon should not hide the length measurement, hence $\Delta x \geq l_{pl}$.

- Refinement: $\Delta t \sum_i \Delta x^i \geq l_{pl}^2$ and $\sum_{i \neq j} \Delta x^i \Delta x^j \geq l_{pl}^2$ (Doplicher, Fredenhagen, Roberts '95). Valid in Quantum Minkowski and in similar fashion in curved space.
- Gravity + QM $\Rightarrow$ *quantization* of spacetime $\rightarrow$ noncommutative geometry (effective theory)
- In nonassociative models: Also volume quantization
- NC and NA emerge naturally in quantization: NC $\leftrightarrow$ non-zero magnetic flux; NA $\leftrightarrow$ non-zero magnetic charge. (Jackiw ' 04)

Apply quantization procedures to spacetime manifold itself

Quantization

- Quantization map $\rho : \mathcal{A}_{\mathrm{CM}} \hookrightarrow \mathcal{A}_{\mathrm{QM}}$ between observable algebras such that classical states $\omega : \mathcal{A}_{\mathrm{CM}} \to \mathbb{C}$ (positive, linear functionals) are classical limits of QM states $\widetilde{\omega} : \mathcal{A}_{\mathrm{QM}} \to \mathbb{C}$.
- Quantization: commutative $\to$ noncommutative
- Mainly use Hamiltonian approach: phase space $\mathcal{P}$, observables $(\mathcal{C}(\mathcal{P}), \{.,.\}_{\mathrm{PB}})$, states $\omega : f \mapsto \int f \mathrm{d}\mu(\omega)$, pure states δ_x. In general $\mathcal{A}_{\mathrm{CM}}$ Poisson-*-algebra.
    - *Canonical:* $\mathcal{A}_{\mathrm{QM}} \overset{\sim}{=} \mathcal{B}(\mathfrak{H})$, ρ defines ordering prescription, e.g. standard or Weyl ordering. Replace PB by commutator $\Leftrightarrow$ non-trivial Lie integration of Poisson to Quantomorphisms.
    - *Deformation:* $\mathcal{A}_{\mathrm{QM}} \overset{\sim}{=} (\mathcal{C}(\mathcal{P})[[\lambda]], *)$. ρ defines $*$-product (connected to ordering and symmetries).
    - *Geometric:* Here Schrödinger picture is used and $\mathfrak{H}$ of states is constructed from symplectic $(\mathcal{P}, \omega)$.
    - *Constraints:* Symmetry vs reduction
    - Points in phase space no longer states in QM! $\Rightarrow$ strict locality lost
- Quantum Spacetime/Noncommutative Geometry: Find proper quantization map for spacetime, e.g. for de Sitter, that reduces to classical spacetime in some classical limit. Then: NC gauge theory, NC gravity, etc.
- For example NC gravity:
    1. Deformed diffeomorphisms
    2. Gravity as NC gauge theory

Examples/Quantization Strategies

- Often: Impose classical symmetries and extend spacetime to larger spacetime including symmetry generators (e.g. Buric, Madore for dS or Steinacker for FLRW.)
- MATRIX MODELS: Often IKKT (reduce SYM to 0D) or BFSS (reduce SYM to 1D). Spacetime $\hat{=}$ spectrum of matrices. Can construct coherent states in this approach. Application of IKKT to cosmology due to H. Steinacker ($\to$ singularity $\hat{=}$ signature change in hyperbolic model [last colloquium at Jacobs])
- DOPLICHER, FREDENHAGEN, ROBERTS '95 (DFR):

$$[\hat{q}^\mu, \hat{q}^\nu] = i\hat{Q}^{\mu\nu}$$
$$[\hat{Q}^{\mu\nu}, \hat{q}^\rho] = 0$$
$$\hat{Q}_{\mu\nu}\hat{Q}^{\mu\nu} = 2(\hat{\vec{m}}^2 - \hat{\vec{e}}^2) = 0$$
$$(\hat{Q}_{\mu\nu}(*\hat{Q})^{\mu\nu})^2 = 4([\hat{\vec{e}} \overset{\bullet}{,} \hat{\vec{m}}]_+)^2 = 16 \cdot \hat{1}$$

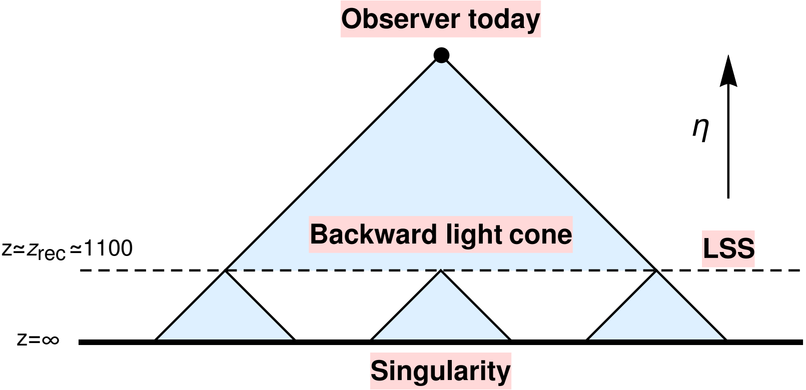
  IMPORTANT PROPERTIES: Unitarity despite space-time NC and Lorentz invariance despite coarse graining
- BTZ BLACK HOLE QUANTIZATION: (Dolan, Gupta, Stern '09) Use geometric identity BTZ $= AdS^3/\sim$, then contruct Poisson brackets compatible with symmetry $\Rightarrow$ $\hat{t}$-spectrum $n\tau - a\tau/2\pi$ where $\alpha = a\tau/(2\pi)$ labels irreps.

Problems of Standard (non-inflationary) Cosmology

- Horizon problem: See next slide
- Flatness problem: $\Omega(t) := \sum_{i=mat,rad,\Lambda} \Omega_i(t) \Rightarrow |\Omega - 1|(t) = \frac{|k|}{(aH)^2(t)}$. Hubble horizon aH is shrinking $\Rightarrow |\Omega - 1| = 0$ unstable point. Again fine-tuning problem: $|\Omega - 1|(t_{pl}) < 10^{-60}$.
- Monopole Problem: (Preskill '79) GUTs produce magnetic monopoles, cosmic evolution too slow to reduce density below current upper bounds.
- Structure formation problem: Non-inflationary evolution does not enhance primordial quantum fluctuations enough to account for fluctuations in CMB and today's structure in universe.

All fine-tuning problems

Horizon Problem – Causal Structure

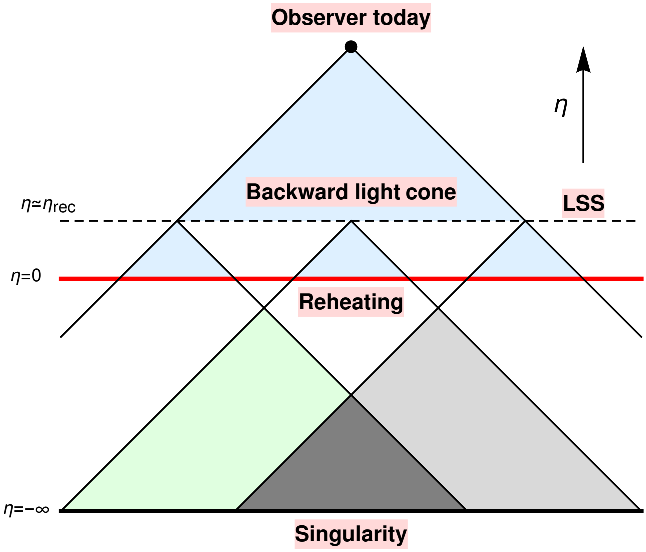


$\mathrm{d}s^2 = a^2(\eta)(-\mathrm{d}\eta^2 + \gamma) = -\mathrm{d}t^2 + a^2(t)\gamma$, $z + 1 = a_0/a$

$\frac{l_{univ}}{l_{LSS}} = \frac{\int_0^{t_0} \frac{\mathrm{d}t'}{a(t')}}{\int_0^{t_{LSS}} \frac{\mathrm{d}t'}{a(t')}} = \frac{\int_0^{a_0} \frac{\mathrm{d}a'}{H(a')(a')^2}}{\int_0^{a_{LSS}} \frac{\mathrm{d}a'}{H(a')(a')^2}}$. for MD $a \sim t^{2/3}$, $H \sim a^{-3/2}$, for RD $a \sim t^{1/2}$, $H \sim a^{-2}$,

hence largest contribution from MD $\Rightarrow \left(\frac{l_{univ}}{l_{LSS}}\right)^2 \approx \left(\frac{\sqrt{a_0}}{\sqrt{a_{LSS}}}\right)^2 = 1 + z_{LSS} \approx 1100$. Fine-tuning problem.

Usual Solution: Inflation – Horizon



Problem: Inflation only accurate down to $t = t_{pl} \Rightarrow$ horizon problem strikes back at some $t \gg t_0$

Time and Length Scales

Time

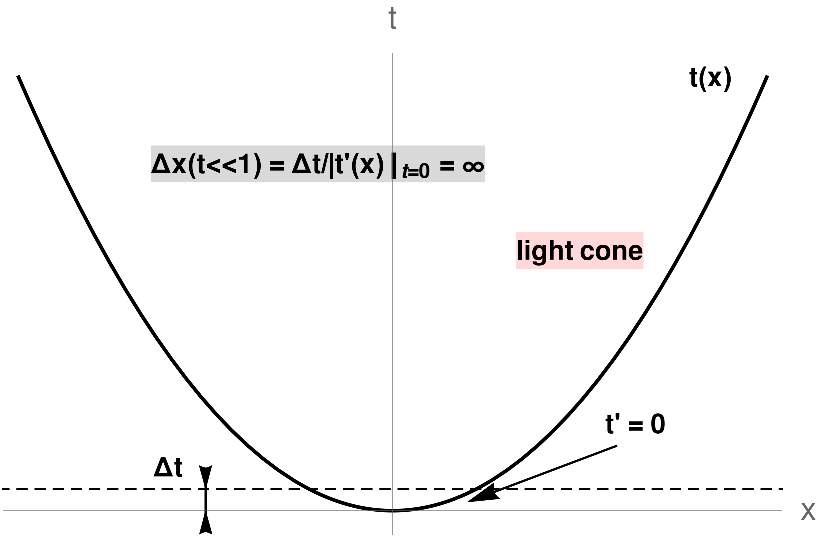
- In general evolution time $\neq$ coordinate time. Consider e.g. Hamiltonian constraint in Canonical QG $\hat{H}|\Psi\rangle = 0$ (Wheeler-deWitt) $\rightarrow$ no explicit time evolution, but statics? No! Relative evolution.
- A priori: entropic time $\neq$ coordinate time $\neq$ relative time

Length Scales

- Consider standard cosmology. QG effects become important when $T \approx T_{pl}$. Then fundamental length scale l_{pl} defined with respect to physical distance $\Delta s \sim l_{pl}$, but often $\Delta x \sim l_{pl}$. But $\Delta x \sim a^{-1}\Delta s$, hence $\Delta x \sim a l_{pl}$?
- Extra dimensions: $l_{pl}^{(4)} = \frac{(l_{pl}^{(D)})^{D/2-1}}{\sqrt{V}}$ (Arkani-Hamed, Dimopoulus, Dvali '98) $\rightarrow$ Planck length might be smaller than it appears.
- Planck's constant might really be operator (Heckmann, Verlinde '15) $[\hat{x}, \hat{p}] \sim \hat{\hbar}$. Define Planck length $\hat{l}_{pl}^2 = \hat{\hbar} G_N / c^3$ such that length scales $\in$ spec($\hat{l}$)?
- Planck mass might be even variable in space and time: variable gravity (Wetterich '14) $M_{pl}^{eff} \sim \chi(t, \vec{x})$ cosmon.

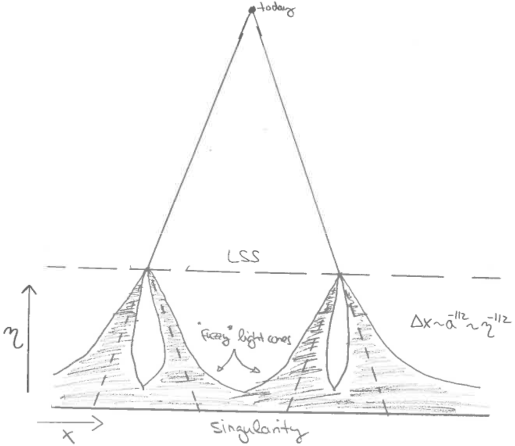
Fuzzy Singularity

Horizon Problem – Initial Time Uncertainty



In radiation domination: Small time uncertainty $\Rightarrow$ infinite space uncertainty

Imposing small time uncertainty at early times leads to fuzzy light cones



"dusty" singularity and "stacked" multiverses. For horizon problem no need of inflation. Problem: How to "average"?

Space-time commutation relations

$$[\hat{t}, \hat{x}^j] = i\lambda\hat{\alpha}^j(t, \vec{r})$$
$$[\hat{\alpha}^j, \hat{x}^k] = [\hat{\alpha}^j, \hat{t}] = 0$$

with $\hat{\vec{\alpha}}$ vector-valued Hermitian operator, that transforms covariantly and is central in the algebra.

Uncertainty: $\Delta x^j \Delta t \geq \frac{\lambda}{2}\alpha^j(t, \vec{r})$ with $\alpha^j := |\langle\hat{\alpha}^j\rangle|$. Lightlike curve satisfies $|\dot{\gamma}_s|(t) = \frac{1}{|a(t)|}$ in comoving coordinates. Impose Δt at some early time $\rightarrow$ geodesic inherits uncertainty $\Delta\gamma_s(t, \vec{r}) = \frac{\Delta t}{a(t)} \geq \frac{\lambda}{2}\frac{\alpha^i(t,\vec{r})}{|a|}\frac{1}{\Delta x}$. If $\Delta x = \Delta\gamma_s \Rightarrow$

Uncertainty

$$\Delta x^i \geq \sqrt{\frac{\lambda\alpha^j}{2|a|}} \overset{\alpha^j \text{ slowly varying}}{\longrightarrow} \infty (t \rightarrow 0)$$

Note: Doplicher, Morsella, Pinamonti '13: Related idea. Using DFR-model, showed that 'localization region' diverges for $t \rightarrow 0$.

Fuzzy Singularity III – Deferred Measurement

How to measure Δt?

Deferred Measurement Principle

In a quantum circuit measurements and operations may be interchanged. $\Rightarrow$ Measurement may be shifted to the end.

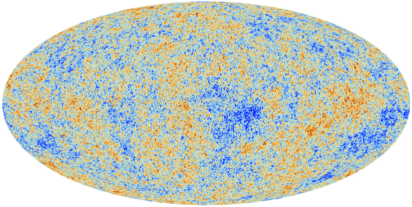
- Measurement: Positive operator valued measure (POVM): $\{\hat{E}_i\}$ self-adjoint operators with $\sum_i \hat{E}_i = \hat{\mathbb{1}}$. Writing $\hat{\rho} = \sum_i \hat{E}_i \hat{\rho} \rightarrow$ measurement $\hat{\rho} \mapsto \hat{E}_i \hat{\rho}$ with probability $\mathbb{P}_i = \mathrm{Tr}(\hat{E}_i \hat{\rho})$.
- Operations/Quantum channel: Completely positive maps between density matrices. In canonical Kraus form: $\hat{\rho} \mapsto \hat{\rho}' = \sum_{i=1}^r \hat{A}_i \hat{\rho} \hat{A}_i^\dagger$ with $\sum_{i=1}^r \hat{A}_i^\dagger \hat{A}_i = \hat{\mathbb{1}}$ and $\mathrm{Tr}(\hat{A}_i^\dagger \hat{A}_j) = d_i \delta_{ij}$, $\sum_{i=1}^r d_i = \dim(\mathcal{H})$ with r the Kraus rank.
- Most general temporal evolution given by Lindblad equation (like above plus two additional terms)

$\Rightarrow$ Strategy: Follow photon with energy uncertainty (induced by time uncertainty), then initial time uncertainty should be detectable much later (e.g. on LSS).

Methods for Investigating Possible Signatures in CMB

CMB

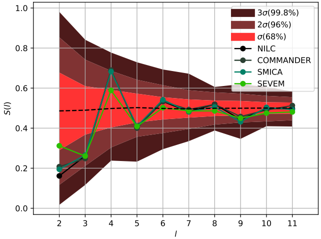
Image from Planck 2015 results (Ade et al. '15)

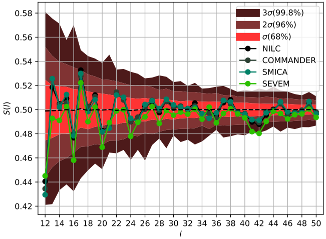


- Relic black body radiation from LSS at about $z \approx 1100$ shortly after recombination
- Small anisotropies depict energy density fluctuations from primordial quantum fluctuations
- Observation on celestial sphere: $\frac{\delta T}{T_0} = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}$ with $T_0 \approx 2.7\,\mathrm{K}$ and $\frac{\delta T}{T_0} \sim 10^{-5} - 10^{-4}$.
- Question: Is δT Gaussian and isotropic random field? Isotropy: (weak/statistical) cosmological principle. Gaussianity: e.g. inflation.
- (1) What are QS signatures in CMB? (2) Are they present in the data?

Fixed multipole: $\{a_{lm}\} \overset{1:1}{\leftrightarrow} \{\vec{v}^{(l,i)} \in \mathbb{RP}^2\}$ depicted as l unit vectors in one hemisphere. Isotropic and Gaussian temperature fluctuations ⇒ MPVs feel repulsion. NOT uniformly distributed.

(Pinkwart, Schwarz; in prep)



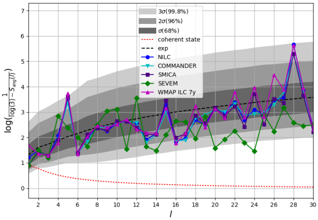


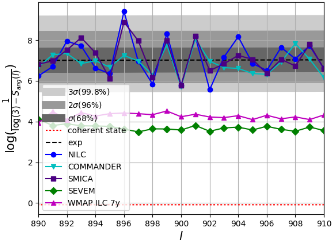
$S(l) = \frac{1}{l}\sum_{i=1}^{l} |\vec{v}^{(l,i)} \cdot \vec{D}|$, $\vec{D}$ cosmic dipole. (Anti-)correlation on large angular scales → nature of dipole not fully understood or other effects?

Pseudo Entropies

Strategy: pure states yield vanishing von Neumann entropy → construct mixed density matrix such that entropy is rotationally invariant.

(Pinkwart, Schupp; in prep)





$S_{ang}(l) = -\mathrm{Tr}(\rho_{ang}(l)\ln(\rho_{ang}(l)))$ with $\rho_{ang}(l) = \frac{1}{l(l+1)}\sum_{i=1}^{3} L_i \rho(l) L_i$, where $\rho(l) = |\Psi_l\rangle\langle\Psi_l|$ and $|\Psi_l\rangle = \sum_{m=-l}^{l} \tilde{a}_{lm}|l,m\rangle$, $\tilde{a}_{lm} = \frac{a_{lm}}{\sum_{n=-l}^{l} a_{ln}}$.

Minimal for Bloch coherent states (?) ; maximal for maximally mixed ρ_{ang}.

— Phenomenological description of QG $\rightarrow$ Quantum geometry

— Quantizing spacetime $\Rightarrow$ Functions on manifold $\rightarrow$ Noncommutative algebra

— Small initial time uncertainty causally connects all initial points

— Possible tools for investigating signatures in CMB: MPVs and pseudo-entropies