I-C universal relation of the scalarized rotating neutron stars with realistic equations of state

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Research Training Group
Graduiertenkolleg
Models of Gravity

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Einstein can not explain everything?!

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Modifying Gravity?

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Modifying Gravity? Maybe!

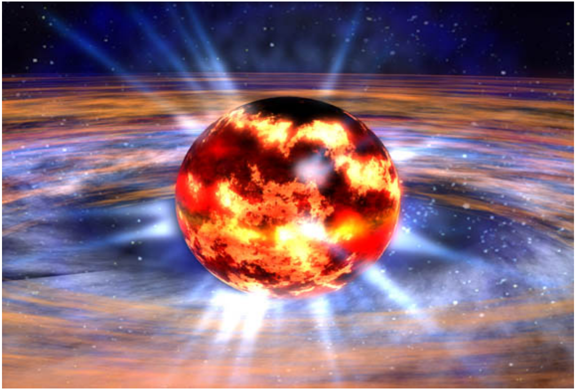
E. Berti *et al.*, "*Testing General Relativity with Present and Future Astrophysical Observations*", Class. Quant. Grav. 32, 243001 (2015).

Scalar-Tensor Theories of gravity (STT)

Natural generalizations of General Relativity:
include **scalar field**, as an additional mediator to the action
besides the **metric** tensor mediator of GR.

Zahra Altaha Motahar, Jose Luis Blzquez-Salcedo, Burkhard Kleihaus, Jutta Kunz, Phys. Rev. D 96, 064046 (2017).

Why Neutron Stars?



so Dense and **so Compact**

$\downarrow$

an ideal laboratory for **Testing Gravity** in the strong field regime.

The gravitational action

- Action in the physical Jordan frame

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-\tilde{g}}\left[F(\Phi)\tilde{\mathcal{R}} - Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2U(\Phi)\right]$$
$$+S_m\left[\Psi_m;\tilde{g}_{\mu\nu}\right]$$

where

$$\tilde{T}_{\mu\nu} = (\tilde{\varepsilon}+\tilde{p})\tilde{u}_\mu\tilde{u}_\nu + \tilde{p}\tilde{g}_{\mu\nu}$$
$$g_{\mu\nu} = F(\Phi)\tilde{g}_{\mu\nu}$$
$$\left(\frac{d\varphi}{d\Phi}\right)^2 = \frac{3}{4}\left(\frac{d\ln(F(\Phi))}{d\Phi}\right)^2 + \frac{Z(\Phi)}{2F(\Phi)}$$
$$A(\varphi) = F^{-1/2}(\Phi)\,,\ 2V(\varphi) = U(\Phi)F^{-2}(\Phi)$$

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- Action in the Einstein frame

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-g}\left[\mathcal{R} - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 4V(\varphi)\right]$$
$$+ S_m[\Psi_m; \mathrm{A}^2(\varphi)g_{\mu\nu}]$$

The field equations in the Einstein frame

Variations with respect the metric tensor lead to the Einstein-matter field equations:

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = +2\partial_\mu \varphi \partial_\nu \varphi - g_{\mu\nu} g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi + 8\pi T_{\mu\nu}$$

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where

$$k(\varphi) = \frac{d\ln(A(\varphi))}{d\varphi}$$

$$T_{\mu\nu} \equiv A^2 \tilde{T}_{\mu\nu}$$

The coupling functions

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- $\beta_1 = -4.5$
- $\beta_2 = -4.8$ (Note: It is already constrained by observation!)

Metric corresponding to a slowly rotating star

$$ds^2 = -e^{f(r)}dt^2 + \frac{1}{n(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta(d\phi + \omega(r)dt)^2$$

ODEs

The Einstein field equations in the slow rotation approximation reduce to the following system of ODEs

$$\frac{dn}{dr} = -\frac{1}{r}\left[8\pi r^2 A^4(\varphi)\tilde{\varepsilon} + nr^2\left(\frac{d\varphi}{dr}\right)^2 + n - 1\right]$$

$$\frac{df}{dr} = \frac{1}{nr}\left[8\pi r^2 A^4(\varphi)\tilde{p} + nr^2\left(\frac{d\varphi}{dr}\right)^2 - n + 1\right]$$

$$\frac{d\tilde{p}}{dr} = -(\tilde{\varepsilon} + \tilde{p})\left[\frac{4\pi r A^4(\varphi)\tilde{p}}{n} + \frac{r}{2}\left(\frac{d\varphi}{dr}\right)^2 + k(\varphi)\left(\frac{d\varphi}{dr}\right) - \frac{n-1}{2nr}\right]$$

ODEs

$$\frac{d^2\omega}{dr^2} = \frac{4\pi r A^4(\varphi)}{n}(\tilde{\varepsilon} + \tilde{p})\left[\left(\frac{d\omega}{dr}\right) + \frac{4(\omega - \Omega)}{r}\right] + \left(\frac{d\omega}{dr}\right)\left[r\left(\frac{d\varphi}{dr}\right)^2 - \frac{4}{r}\right]$$

$$\frac{d^2\varphi}{dr^2} = \frac{4\pi r A^4(\varphi)}{nr}\left[r\left(\frac{d\varphi}{dr}\right)(\tilde{\varepsilon} - \tilde{p}) + k(\varphi)(\tilde{\varepsilon} - 3\tilde{p})\right] - \left(\frac{d\varphi}{dr}\right)\frac{(n+1)}{nr}$$

Expansion at the center

Expansion at the center of the star in terms of the radial coordinate:

$$m(r) = \frac{4}{3}\pi A_0^2 \tilde{\varepsilon}_0 r^3 + O(r^4)$$

$$f(r) = f_0 + \frac{4}{3}\pi A_0^2 (\tilde{\varepsilon}_0 + 3\tilde{p}_0) r^2 + O(r^3)$$

$$\tilde{p}(r) = \tilde{p}_0 - \frac{1}{6}\pi(\tilde{\varepsilon}_0 + \tilde{p}_0)\left[4A_0(\tilde{\varepsilon}_0 + 3\tilde{p}_0) + (A_0')^2(\tilde{\varepsilon}_0 - 3\tilde{p}_0)\right] r^2 + O(r^3)$$

$$\omega(r) = \omega_0 - \frac{8}{5}\pi A_0^4(\Omega - \omega_0)(\tilde{\varepsilon}_0 + \tilde{p}_0) r^2 + O(r^3)$$

$$\varphi(r) = \varphi_0 + \frac{1}{3}\pi A_0 A_0' (\tilde{\varepsilon}_0 - 3\tilde{p}_0) r^2 + O(r^3)$$

Expansion at infinity

If we requiere the solution to be asymptotically flat, then close to infinity the functions satisfy the following behaviour:

$$m(r) = M - \frac{1}{2}\frac{\omega_A^2}{r} - \frac{1}{2}\frac{\omega_A^2 M}{r^2} + +O(\frac{1}{r^3})$$

$$f(r) = -\frac{2M}{r} - \frac{2M^2}{r^2} - \frac{1}{3}\frac{M(M^2 - \omega_A^2)}{r^3} + O(\frac{1}{r^4})$$

$$\varphi(r) = \frac{\omega_A}{r} + \frac{M\omega_A}{r^2} + \frac{1}{6}\frac{\omega_A(8M^2 - \omega_A^2)}{r^3} + O(\frac{1}{r^3})$$

$$\omega(r) = \frac{2J}{r^3} + O(\frac{1}{r^5})$$

$$I = J/\Omega$$

EOS

In order to integrate the system we have to provide an equation of state in the form $\tilde{\varepsilon} = \tilde{\varepsilon}(\tilde{p})$.

- **Polytropic EOS:** $\tilde{\varepsilon} = K\frac{\tilde{\rho}^{\Gamma}}{\Gamma-1} + \tilde{\rho}$, $\tilde{p} = K\tilde{\rho}^{\Gamma}$, with $\tilde{\rho}$ being the baryonic mass density.

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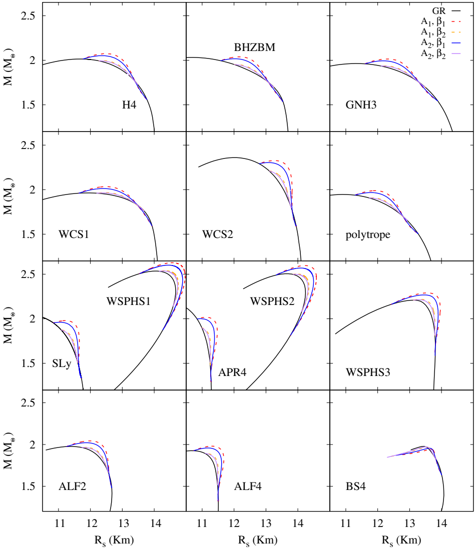
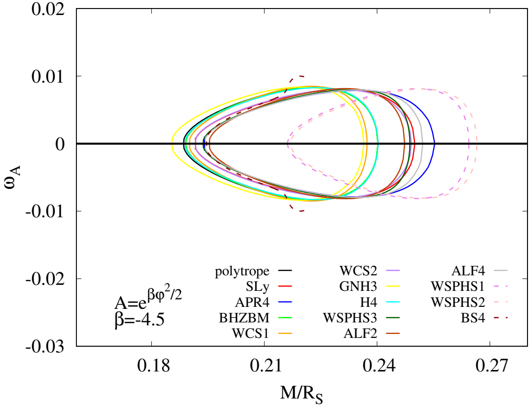
- **Realistic EOS:** Two EOSs containing just *nuclear matter*: SLy and APR4.
  Five EOSs containing *nucleons+hyperons*: BHZBM, GNH3, H4 and WCS1-2.
  Two EOSs for *pure quark matter*: WSPHS1 and 2.
  Four EOSs containing *hybrid quark+nucleons*: ALF2-4, BS4 and WSPHS3.

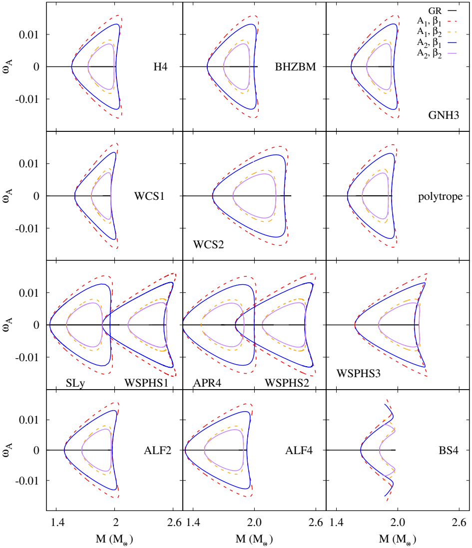
Total Mass versus the physical Radius of the Neutron Stars



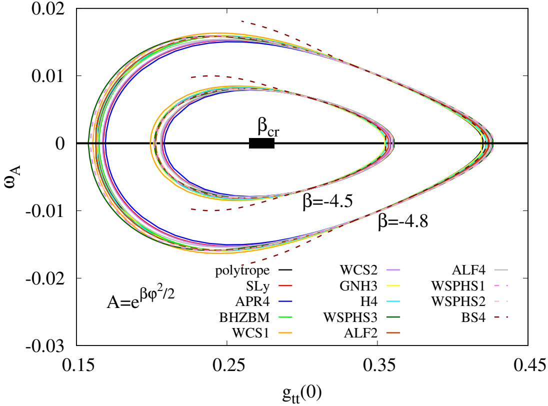
Scalar field charge ω_A versus the Compactness $\mathcal{C} = M/R_s$ of the Neutron Star models for various EOSs



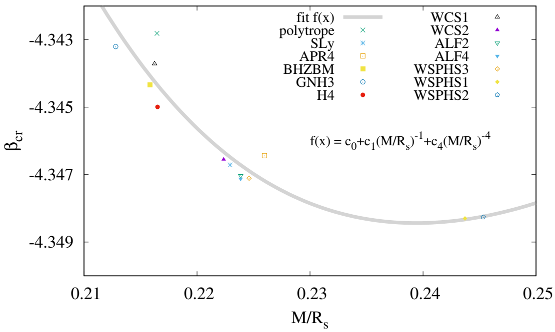
Scalar field charge versus the total Mass of the Neutron Stars, for various EOS.



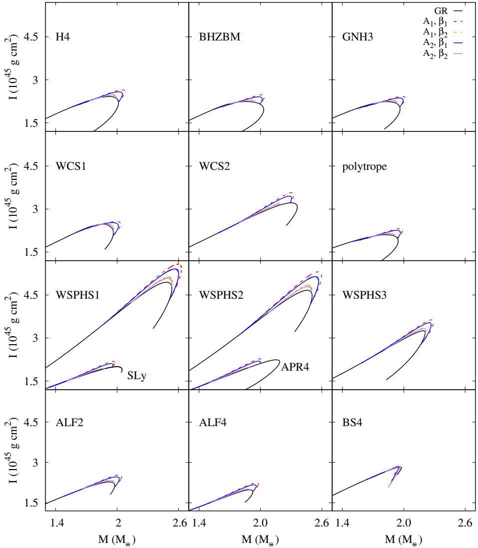
Scalar field charge versus the $g_{tt}(0)$, for various EOSs.



Onset of scalarization β_{crit} vs Compactness fit function



Moment of inertia versus total Mass of the Neutron Stars



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- Multipole moments 3-Hair
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Daniela D. Doneva & George Pappas, "*Universal Relations and Alternative Gravity Theories*", (23 Sep 2017), arXiv:1709.08046v1.

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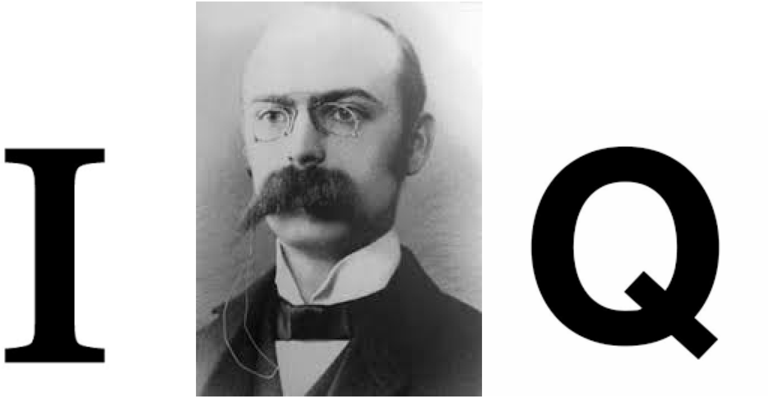

Kent Yagi & Nicolas Yunes, (2013), arXiv:1302.4499 [gr-qc].

Universal I-$\mathcal{C}$ Relations

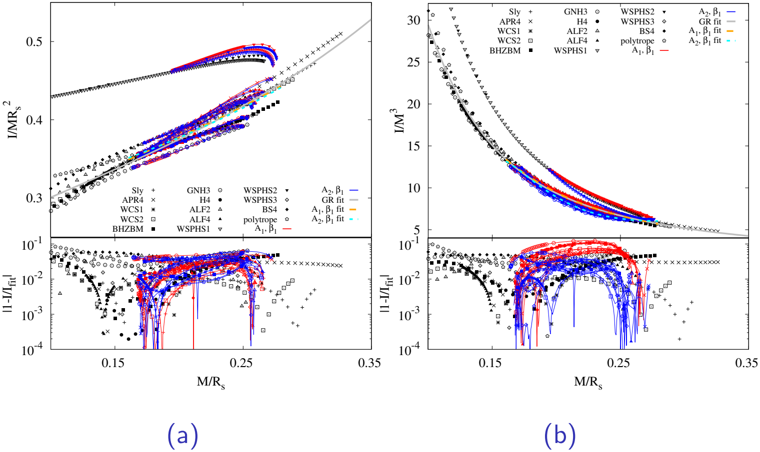

Figure: Moment of Inertia versus Compactness for different normalizations. In (a), the moment of inertia is scaled to MR_s^2, while in (b) we scale it to M^3.

QNMs of Neutron Stars in STT

Gravitational Waves: a new window into the universe

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- Constructing compact star models for a broaden spectrum of alternative theories of gravity.