TESTS OF GRAVITY
WITH GRAVITATIONAL WAVES & COSMOLOGY
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OUTLINE

• The EFT of cosmological perturbations.
• The impact of GW170817.
• Constraints from cosmology & summary.
THE EFT OF COSMOLOGICAL PERTURBATIONS
Let $\Theta$ be a vector of fields mediating gravity.

2. Taylor expand the gravitational Lagrangian in perturbations $\delta\Theta$:

$$L \simeq \bar{L} + L_{\Theta_a} \delta \Theta_a + \frac{1}{2} L_{\Theta_a \Theta_b} \delta \Theta_a \delta \Theta_b + \ldots$$

This gives us linearised gravitational field equations.

Scalar perturbations of metric:

$$ds^2 = - (1 + 2\Phi) dt^2 + 2 \partial_i B \, dx^i \, dt$$

$$+ a^2(t) \left[ (1 - 2\Psi) \delta_{ij} + 2 \partial_i \partial_j E \right] \, dx^i \, dx^j$$
3. Build Lagrangian containing all combinations of fields in $\Theta$. Simplest case: $\Phi, \Psi, B, E, \phi$.

$$\delta_2 L = L_{\Phi\Phi} \Phi^2 + L_{\Phi\Psi} \Phi \Psi + \ldots$$

$$+ L_{\Psi\Psi} \dot{\Psi}^2 + L_{\Psi \partial^2 B} \dot{\Psi} \partial_i \partial^i B + \ldots$$

$$+ L_{\phi\phi} \delta \dot{\phi}^2 + L_{\phi \partial^2 E} \delta \phi \partial_i \partial^i \dot{E} + \ldots$$

$$+ \text{usual fluid matter sector}$$

$\sim 70$ terms in total.
SYMMETRIES AND CONSEQUENCES

4. The action must be coordinate-invariant.

⇒ Enforce linear diff symmetry: \( x^\mu \to x^\mu + \epsilon^\mu \)
\[ \epsilon^\mu = (\pi, \partial^i \epsilon) \]

⇒ \( \delta_2 S \to \delta_2 S + \left[ \text{terms linear in } \delta \phi, \phi, \psi \text{ etc.} \right] \times (\pi \text{ or } \epsilon) \)

must vanish

Invariance of action under a non-dynamical symmetry gives a set of constraint relations.
SYMMETRIES AND CONSEQUENCES

4. The action must be coordinate-invariant.

Invariance of action under a non-dynamical symmetry gives a set of constraint relations.

\[ \delta_2 L = L_{\Phi \Phi} \Phi^2 + L_{\Phi \Psi} \Phi \Psi + \ldots \]

\[ + L_{\dot{\Psi} \dot{\Psi}} \dot{\Psi}^2 + L_{\dot{\Psi}} \partial^2 B \dot{\Psi} \partial_i \partial^i B + \ldots \]

\[ + L_{\phi \phi} \delta \phi^2 + L_{\phi} \partial^2 \dot{E} \delta \phi \partial_i \partial^i \dot{E} + \ldots \]

\[ + \text{usual fluid matter sector} \]
SYMMETRIES AND CONSEQUENCES

4. The action must be coordinate-invariant.

Invariance of action under a non-dynamical symmetry gives a set of constraint relations. E.g.:

\[ L_{\Phi \Phi} + 3\dot{H} L_{\Phi \Psi} = 0 \]

\[ L_{\phi \phi} - L_{\dot{\psi} \dot{\psi}} + 5 L_{\phi \partial^2 \dot{E}} = 0 \quad \text{etc.} \]

⇒ Solve system constraint equations.
⇒ Reduce to a handful of `true’ free functions.
THE ALPHA PARAMETERS

\( \alpha_T(t) \): speed of gravitational waves, \( c_T^2 = 1 + \alpha_T \).

\( \alpha_K(t) \): kinetic term of scalar field.

\( \alpha_B(t) \): ‘braiding’ – mixing of scalar + metric kinetic terms.

\( \alpha_M(t) = \frac{1}{H} \frac{d \ln M^2(t)}{dt} \): running of effective Planck mass.

\( \alpha_H(t) \): disformal symmetries of the metric.
# The Alpha Parameters

<table>
<thead>
<tr>
<th>Model Class</th>
<th>$\alpha_K$</th>
<th>$\alpha_B$</th>
<th>$\alpha_M$</th>
<th>$\alpha_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda CDM$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>quintessence</td>
<td>$(1 - \Omega_m)(1 + w_X)$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$k$-essence/perfect fluid</td>
<td>$\frac{(1 - \Omega_m)(1 + w_X)}{c_s^2}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>kinetic gravity braiding</td>
<td>$\frac{m^2(n_m + \kappa_\phi)}{H^2 M_{Pl}^2}$</td>
<td>$\frac{m \kappa}{H M_{Pl}^2}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>galileon cosmology</td>
<td>$-\frac{3}{2} \alpha_M^3 H^2 t_c^2 e^{2\phi/M}$</td>
<td>$\frac{\alpha_K}{6} - \alpha_M$</td>
<td>$-\frac{2\dot{\phi}}{HM}$</td>
<td>0</td>
</tr>
<tr>
<td>BDK</td>
<td>$\frac{\dot{\phi}^2 K^{\phi}_{,\phi} e^{-\kappa}}{H^2 M^2}$</td>
<td>$-\alpha_M$</td>
<td>$\frac{\dot{\phi}}{H}$</td>
<td>0</td>
</tr>
<tr>
<td>metric $f(R)$</td>
<td>0</td>
<td>$-\alpha_M$</td>
<td>$\frac{B H}{H^2}$</td>
<td>0</td>
</tr>
<tr>
<td>MSG/Palatini $f(R)$</td>
<td>$-\frac{3}{2} \alpha_M^2$</td>
<td>$-\alpha_M$</td>
<td>$\frac{2\dot{\phi}}{H}$</td>
<td>0</td>
</tr>
<tr>
<td>$f$(Gauss-Bonnet)</td>
<td>0</td>
<td>$\frac{-2H \dot{\xi}}{M^2 + H \xi}$</td>
<td>$\frac{H \dot{\xi} + H \dot{\xi}}{H(M^2 + H \xi)}$</td>
<td>$\frac{\dot{\xi} - H \dot{\xi}}{M^2 + H \xi}$</td>
</tr>
</tbody>
</table>
VECTOR-TENSOR RESULTS

We can play the game over again with a vector field.

\[ A^\mu \sim (\bar{A} - \delta A_0, \partial^i \delta A_1) \]

Results:
- \( \alpha_T(t), \alpha_K(t), \alpha_M(t) \) as for the scalar case.
- \( \alpha_V(t) \) vector mass mixing \( \sim \delta A_0 \delta A_1 \)
- \( \alpha_D(t) \) small-scale dynamics \( \sim k_i^4 \delta A_1^2 \)
- \( \alpha_A(t) \) `auxiliary friction' \( \sim \delta A_0 \delta \dot{A}_1 \)
- \( \alpha_C(t) \) conformal coupling excess \( \sim \) changes effective mass scale of theory
## RESULTS BREAKDOWN

<table>
<thead>
<tr>
<th>Fields</th>
<th># True free functions</th>
<th>Example full theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{\mu\nu}$</td>
<td>1</td>
<td>GR</td>
</tr>
<tr>
<td>$g_{\mu\nu}, \phi$</td>
<td>5</td>
<td>Horndeski</td>
</tr>
<tr>
<td>$g_{\mu\nu}, A^\mu$</td>
<td>7</td>
<td>Generalised Proca</td>
</tr>
<tr>
<td>$g_{\mu\nu}, A^\mu, \lambda$</td>
<td>4</td>
<td>Einstein-Aether</td>
</tr>
<tr>
<td>$g_{\mu\nu}, q_{\mu\nu}$</td>
<td>5</td>
<td>Massive Bigravity</td>
</tr>
<tr>
<td>$g_{\mu\nu}, \phi, A^\mu, q_{\mu\nu}$</td>
<td>12</td>
<td>Uber-case</td>
</tr>
</tbody>
</table>

(Some restrictions imposed, e.g. 1 propagating d.o.f.)
THE IMPACT OF GRAVITATIONAL WAVES
**THE ALPHA PARAMETERS**

\[ \alpha_T(t) : \text{ speed of gravitational waves, } c_T^2 = 1 + \alpha_T. \]

\[ \alpha_K(t) : \text{ kinetic term of scalar field.} \]

\[ \alpha_B(t) : \text{ ‘braiding’ – mixing of scalar + metric kinetic terms.} \]

\[ \alpha_M(t) = \frac{1}{H} d \ln \frac{M^2(t)}{dt} : \text{ running of effective Planck mass.} \]

\[ \alpha_H(t) : \text{ disformal symmetries of the metric.} \]
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\[ c_T^2 = 1 + \alpha_T \]

\[ \Delta t \simeq 1.7 \text{ s} \]

\[ \Rightarrow |\alpha_T| \lesssim 10^{-15} \]

!! c. f. \[ |\alpha_M|, |\alpha_B| \lesssim \mathcal{O}(1) \]

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Assumptions:

1. No fine-tuned cancellation of intrinsic emission delay.

2. Propagation time dominated by cosmological regime.

3. No finely-tuned, protected functional cancellations in theory.
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What does this mean for gravity theories?

Scalar case clearest; full theory is **Horndeski gravity**.

\[
S = \int d^4x \sqrt{-g} \sum_{i=2}^{5} \mathcal{L}_i + S_M
\]

\[
\mathcal{L}_2 = K
\]

\[
\mathcal{L}_3 = -G_3 \Box \phi
\]

\[
\mathcal{L}_4 = G_4 R + G_{4,X} \left\{ (\Box \phi)^2 - \nabla_\mu \nabla_\nu \phi \nabla^\mu \nabla^\nu \phi \right\}
\]

\[
\mathcal{L}_5 = G_5 G_{\mu \nu} \nabla^\mu \nabla^\nu \phi - \frac{1}{6} G_{5,X} \left\{ (\nabla \phi)^3 \right. \\
\left. - 3 \nabla^\mu \nabla^\nu \phi \nabla_\mu \nabla_\nu \phi \Box \phi + 2 \nabla^\nu \nabla_\mu \phi \nabla^\alpha \nabla_\nu \phi \nabla^\mu \nabla_\alpha \phi \right\}
\]

where \( G_i = G_i (\phi, X) \) and \( X = -\nabla_\nu \phi \nabla^\nu \phi / 2 \).
GW170817 & GRB 170817A

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\[ \mathcal{L}_5 = G_5 G_{\mu \nu} \nabla^\mu \nabla^\nu \phi - \frac{1}{6} G_{5,X} \left\{ (\nabla \phi)^3 \right\} \]

Linearised theory maps to alpha parameters:

\[ \Rightarrow \alpha_T(t) = \frac{2X}{M_*^2} \left[ 2G_{4,X} - 2G_{5,\phi} - \left( \ddot{\phi} - \dot{\phi} H \right) G_{5,X} \right] \]

Barring fine-tuned cancellations, \( \Rightarrow G_{4,X} = G_{5,\phi} = G_{5,X} = 0 \).
GW170817 & GRB 170817A

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\[ \mathcal{L}_4 = G_4 R \]

\[ \mathcal{L}_5 = G_5 G_{\mu\nu} \nabla^\mu \nabla^\nu \phi = 0 \quad \text{by Bianchi identity} \]

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GW170817 & GRB 170817A

What does this mean for gravity theories?

Scalar case clearest; full theory is **Horndeski gravity**.

\[
\mathcal{L}_4 = G_4 R \\
\mathcal{L}_2 = K \\
\mathcal{L}_3 = -G_3 \Box \phi
\]

\[
\phi = f_R \\
f(R) - R f_R
\]

⇒ f [R] gravity fits the template, so it survives.
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What does this mean for gravity theories?

The vector-tensor equivalent of Horndeski is **Generalised Proca**: 

$$\mathcal{L}_1 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad \mathcal{L}_2 = G_2 \quad \mathcal{L}_3 = G_3 \nabla_\mu A^\mu$$

$$\mathcal{L}_4 = G_4 R + G_{4,X} \left[ (\nabla_\mu A^\mu)^2 + c_2 \nabla_\rho A_\sigma \nabla^\rho A^\sigma - (1 + c_2) \nabla_\rho A_\sigma \nabla^\sigma A^\rho \right]$$

$$\mathcal{L}_5 = G_5 G_{\mu\nu} \nabla^\mu A^\nu - \frac{1}{6} G_{5,X} \left[ (\nabla_\mu A^\mu)^3 - 3d_2 \nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\rho A^\sigma + \text{similar} \right]$$

where \( G_i = G_i(X) \) and \( X = -\frac{1}{2} A_\mu A^\mu \).
GW170817 & GRB 170817A

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\]

\[
\mathcal{L}_5 = G_5 G_{\mu\nu} \nabla^\mu A^\nu \\
- \frac{1}{6} G_{5,X} \left[ (\nabla_\mu A^\mu)^3 - 3d_2 \nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\rho A^\sigma + \text{similar} \right]
\]

\[
\alpha_T = \frac{A^2}{\tilde{M}^2_*} \left[ 2G_{4,X} - (HA - \dot{A})G_{5,X} \right] \quad \Rightarrow \quad G_{4,X} = G_{5,X} = 0
\]
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\[
\mathcal{L}_5 = G_5 G_{\mu \nu} \nabla^\mu A^\nu = 0
\]

\[
\alpha_T = \frac{A^2}{\tilde{M}_*^2} \left[ 2G_{4,X} - (HA - \dot{A})G_{5,X} \right] \implies G_{4,X} = G_{5,X} = 0
\]

by Bianchi identity again.
GW170817 & GRB 170817A

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The vector-tensor equivalent of Horndeski is **Generalised Proca**:

\[
\mathcal{L}_1 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad \mathcal{L}_2 = G_2 \quad \mathcal{L}_3 = G_3 \nabla_\mu A^\mu \\
\mathcal{L}_4 = G_4 R
\]
GW170817 & GRB 170817A

What does this mean for gravity theories?

For bimetric theories, get a bound on graviton mass:

\[ m_g \lesssim 10^{-22} \text{eV} \]

This is not competitive with existing Solar System bounds:

\[ m_g \lesssim 10^{-32} \text{eV} \]

(from Lunar Laser Ranging & Earth-Moon precession)
COSMOLOGICAL OBSERVATIONS
THE ALPHA PARAMETERS

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$\alpha_H(t)$: disformal symmetries of the metric.
CODES

Effective Field Theory with CAMB


Zumalacarregui, Bellini, Sawicki & Lesgourgues (2016).
EFFECTS ON OBSERVABLES

CMB temperature power spectrum.

CMB lensing power spectrum
THE CURRENT STATE OF PLAY

\[ \alpha_B(z) = \alpha_0^B \alpha^\xi \]

\[ \frac{\delta M(z)}{M_{Pl}} = \tilde{M}_0 \alpha^\beta \]

SDSS (galaxy survey) + Planck CMB + BOSS BAOs & RSDs + lensing data.

Kreisch & Komatsu, 1712.02710
THE CURRENT STATE OF PLAY

\[
\frac{\delta M(z)}{M_{Pl}} = \tilde{M}_0 a^\beta
\]

\[
\alpha_B(z) = \alpha_0^B a^\xi
\]

Caution: stability conditions lead to non-trivial contours.

Kreisch & Komatsu, 1712.02710
ONGOING & FUTURE EXPERIMENTS

- Dark Energy Survey (ongoing)
- Euclid (2020)
- LSST (2021)
- SKA (2023)
THE NEW THEORY LANDSCAPE
Uncertain? multiscalar-tensor, nonlocal gravity, Chaplygin gases.
Uncertain?  multiscalar-tensor, nonlocal gravity, Chaplygin gases.
CONCLUSIONS

1. The EFT of cosmological perturbations — agnostic & efficient tests of the gravity model landscape.

2. Framework can be linked directly to recent GW events with powerful results.

3. Goal: EFT used by next-generation experiments as standard format for dark energy constraints.

References:
1604.01396
1710.06394
1711.09893