Reduction and Coherence

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Outline

There are various reductive projects.

Some of them are best understood as being concerned with *coherence* between two or more theories, models or other forms of descriptions.

Acknowledging the diversity of reductive projects as well as the concern with coherence helps us to better understand some controversial cases of reduction.

I. Logical Empiricism
II. Theory-reduction (and coherence)
III. Reductive Explanations
IV. Failures of reduction (and coherence)
V. Controversial cases

I. Logical Empiricism

Logical Empiricism and Reduction

For the Logical Empiricists the concept of reduction was closely associated with those of *Unity of Science* and of *Progress*:

“The label ‘reduction’ has been applied to a certain type of progress in science.”

“replacement of an accepted theory … by a new theory … which is in some sense superior to it” (Kemeny&Oppenheim (1956, 6/7)).

“We … think the assumption that unitary science can be attained through cumulative micro-reduction recommends itself as a working hypothesis” (Oppenheim, Putnam 1958, 8)

Logical Empiricism and Reduction

Micro-reduction:

"The essential feature of a micro-reduction is that the branch B_1 deals with the parts of the objects dealt with by B_2. … Under the following conditions we shall say that the reduction of B_2 to B_1 is a micro-reduction: B_2 is reduced to B_1; and the objects in the universe of discourse of B_2 are wholes which possess a decomposition into proper parts all of which belong to the universe of discourse of B_1." (Oppenheim, Putnam 1958, 6)

Logical Empiricism and Reduction

Micro-reduction involves (at least) the following claims:

(1) Reduction is a type of *progress* towards the *Unity of Science*.

(2) Reduction concerns the relation of *theories*.

(3) Reduction involves *derivation*.

(4) Reduction involves *explanation*.

(5) Reduction concerns the *part-whole-relation*.

Dis-entanglement

[(1) Reduction is a type of progress towards the Unity of Science.]

A. Theory-reduction:

(2) Reduction concerns the relation of theories.

(3) Reduction involves derivation.

B. Reduction as part-whole-explanation:

(4) Reduction involves explanation.

(5) Reduction concerns the part-whole-relation.

II. Theory-reduction

Theory-reduction: Nagel

Ernest Nagel's conception of reduction:

"… a reduction is effected, when the experimental laws of the secondary science (…) are shown to be logical consequences of the theoretical assumptions (…) of the primary science." (Nagel 1965, 352)

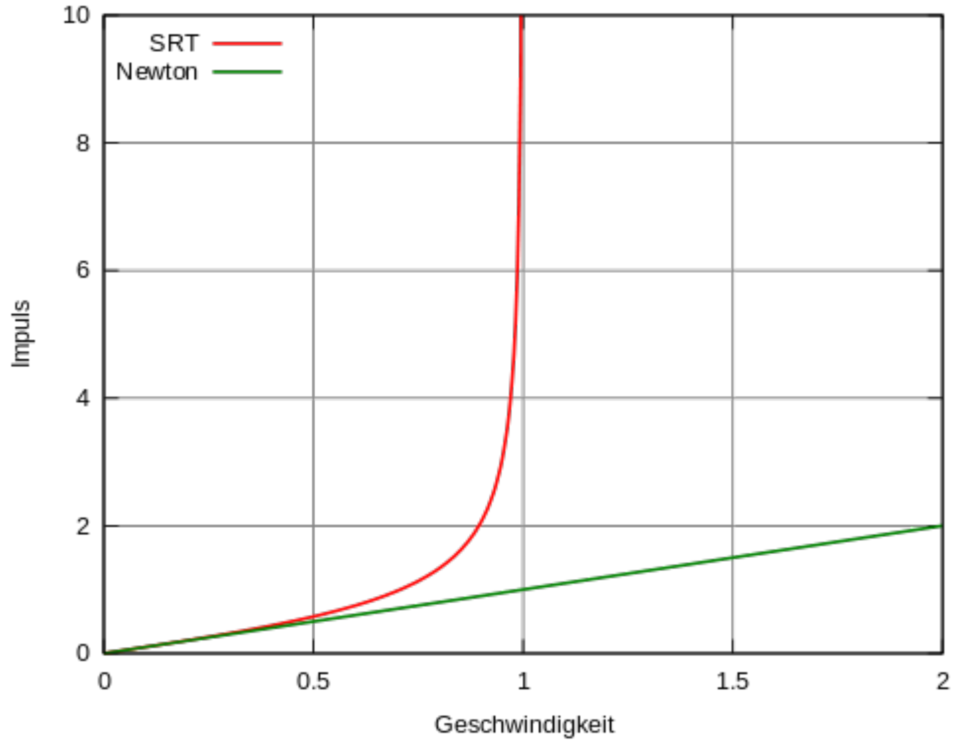
Two formal conditions:

- Condition of connectability
- Condition of derivability

→ If Nagel-reduction succeeds the old theory is integrated completely/ embedded into the new theory.

Theory-reduction: Nagel

Problem: Incompatibility of T_1 and T_2: Nagel reduction does *not* apply in paradigmatic cases, e.g. because SRT and NM predictions differ.



Nagel works nowhere.

Theory-reduction: Limit-case Reduction

→ CM cannot be logically derived from SRT (Nagel reduction only works for cases of complete retention)

However: There is a different sense of reduction according to which the new, refined theory T_n is said to be *reducible*$_2$ to the older theory T_O. (Nickles 1973)

$$\lim_{\varepsilon \to 0} T_N = T_O$$

ε is a parameter appearing in T_N .

Theory-reduction: Limit-case Reduction

Nickles: "epitomizing [the reduction$_2$ of SRT to NM] is the reduction of the Einsteinian formula for momentum,

$$p \;=\; m_0 v \,/\, \sqrt{(1 - (v/c)^2)}$$

where m_0 is the rest mass, to the classical formula $p = m_0 v$ in the limit as $v \rightarrow 0$." (Nickles, 1973, p. 182)

In this context 'reduction$_2$' means: you can skip the complexities of SRT and work with the simpler old theory NM, given certain limiting conditions.

Theory-reduction: Limit-case Reduction

Some remarks:

Rohrlich: "The point is, …, that if a mature coarse theory S is superseded by a finer theory T in such a way that S survives within a certain domain D (and thus becomes an established theory) then it is a *necessary* requirement for vertical logical coherence of theories (…), that such a reduction exists. This means that there must exist a parameter p which permits the above limit to be carried out." (Rohrlich 1988, 305)

→ Rationale for limit case reduction: vertical coherence: We want to understand why the old theory was empirically successful within a certain range and why we can continue to use it within certain confines.

Theory-reduction: Limit-case Reduction

Even though if we had Nagel-reduction we would have limit case reduction too, in the typical case these concepts describe *very different* situations:

- Limit-case reduction is typically *piece-meal*: It might be possible for one pair of equations (from SRT and NM) to be related by a reduction_2-relation while another pair of equations fails in this respect.

- Derivation: Nagel: The laws of the old theory have to be *logically deducible* from the new theory. - Limit-case reduction: The classical equation is derived from the SR-equation by *taking a limit-process*. (Ehlers, Scheibe, Batterman, Fletcher)

- The new theory does not have to *explain* the old one.

III. Reductive Explanations

Reductive Explanations

Assumption:

The issue of what is an explanation and whether an explanation is reductive can be separated.

Observation concerning the use of the expressions "reductive explanation":

Explanations are called "reductive" when they conform to certain additional constraints – when the factors picked out in the explanandum fulfill additional criteria, which are implicitly accepted within a context of research.

These constraints usually concern the *nature* of the explaining factors.

Two cases:

- Causal reductive explanations
- Part-whole explanations

Reductive causal Explanations

Causal explanations are *reductive explanations* if the relevant causal factors picked out are *all of one kind* (this is how the term "reductive explanation" is used):

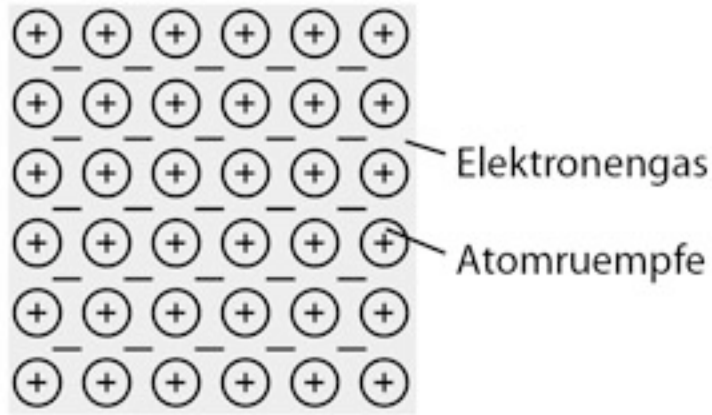
- explanation of human behaviour *exclusively* in terms of evolutionary shaped mental modules (evolutionary psychology)
- explanation of the history of nations *exclusively* in terms of economical factors
- explanation of traits of organisms *exclusively* in terms of genes

→ Reductive causal explanations imply that factors that are of a different kind than the kind in question are causally irrelevant.

Part-whole Explanations

Constitutive explanation (part-whole-explanation)

Example: Electrical and thermal properties of a piece of metal are explained in terms of ions and electrons.



Part-whole Explanations

Example: Classical treatment of the ideal crystal. Explanation of the dynamics (temporal evolution) of a compound system exclusively in terms of the dynamics of the parts and their interactions.

$$H = \sum_i E^i_{kin} + (1/2) \sum_{ij} U_{ij} q_i q_j$$

where $E^i_{kin} = p^2_i/2m$ is the kinetic energy of the parts, and $U_{ij} = \partial^2/\partial q_i \partial q_j U(q_1 .. q_{3N})$ describes the interactions between the parts.

→ permits us to calculate the behavior of the compound system, including measurable thermodynamic properties such as the specific heat c_v:

$$c_v = (\partial/\partial T)\, u(H)$$

Part-whole Explanations

→ Part-whole explanations are reductive in (at least) two respects:

(a) The explanation refers exclusively to the parts, their properties and their interactions.

(b) Very often the properties of the parts are of a particular restricted kind, excluding properties of the kind to be explained (e.g. statistical mechanical properties vs. TD-properties)

→ Pure part-whole explanations are *by definition* reductive in the sense of (a)

→ Part-whole-explanations may invoke theories but need not. (If they invoke theories: parts and compound may be described by the same theory or by different theories)

IV. Failures of Reductive Explanations?

Failures of Reductive Explanations?

1) Causal reductive explanations

What happens if reductive causal explanations fail?

There are causally relevant factors of a *different* type (than the ones envisaged).

→ a causally reductive explanation will be replaced by a different explanation that appeals to a diverse set of causal factors.

So what happens if reduction fails?

→ a reductive causal explanation is replaced by a non-reductive causal explanation.

Failures of Reductive Explanations?

2) part-whole explanations

Case 1:

Reductive Cartesian Program: Explanation of the behaviour of all compound systems in terms of the size, figure and motion of their parts (plus laws of collision).

However, all attempts to micro-explain the behaviour of the solar system (plus some comets) in terms of Cartesian physics have failed.

The attempt to provide a reductive explanation failed.

Failures of Reductive Explanations?

One option would have been so say:

"Gravitational phenomena (such a the observed paths of the planets) cannot be explained in terms of the parts' properties and the laws that apply to them. This is a case of *emergence*. And that is what we have to accept."

[Emergence: There is no explanatory account of how the compound's behaviour can be explained in terms parts' behaviour, i.e. in terms of (i) general laws concerning the behaviour of the components considered in isolation (ii) general laws of composition and (iii) general laws of interaction (C.D. Broad).

→ This concept is different from the use in "Autumn School on Correlated Electrons: Emergent Phenomena in Correlated Matter", Jülich, Sept. 2013).]

Failures of Reductive Explanations?

But this is not what happened:

Physicists redefined the basis on which part-whole-explanation could rely. The constituents were now conceived of as possessing an additional property: mass, along with an additional law that describes the relevance of this property.

Clearly the original reductionist hypothesis had to be given up. But from our perspective this is not a case of emergence, because with the redefined basis the behaviour of the compound can be explained in terms of the properties of the parts, their interactions etc. (see Hoyningen-Huene 1994 for discussing this example in this context)

→ One putative reductive explanation is replaced by a different (presumably more successful) reductive explanation. The constraints on the explanans have changed.

Failures of Reductive Explanations?

Case 2:

Late 19th Century: There was no consistent mechanical model that would yield the observed specific heats of polyatomic gases. (Darrigol and Renn 2013)

"Here we are brought face to face with the greatest difficulty which the molecular theory has yet encountered." (Maxwell)

Reduction seems to fail. One option would have been so say:

"Specific heats of polyatomic gases cannot be explained in terms of the parts' properties plus the laws that apply to them. This is a case of emergence. And that's fine."

Failures of Reductive Explanations?

But this is not what happened:

Physicists redefined the basis on which part-whole-explanation could rely. The constituents were described in terms of quantum statistical mechanics rather than in terms of CSM.

Clearly the original reductionist hypothesis had to be given up. But again: from our perspective it is not a case of emergence, because with the redefined basis the behaviour of the compound can be explained in terms of the properties of the parts, their interactions etc.

→ One putative reductive explanation is replaced by a different (successful) reductive explanation. The constraints on the explanans have changed.

Failures of Reductive Explanations?

A case of emergence would be considered as an *anomaly*.

→ pressure to come along with a different explanatory account of how parts and wholes are connected.

→ pressure for *coherence*:

Our stories about parts and wholes have to fit together: The laws concerning the parts and their interactions need to explain the laws for the compounds (or replace them by laws that are equally well supported by the evidence). That's part of what constitutes good research. (*part-whole-coherence*).

Failures of Reductive Explanations?

Will we ever encounter cases of emergence?

Preferred strategy:

Replace putative reductive explanation by another (more successful) reductive explanation.

This seems to work often because the reductive strategy is flexible:

1) We can revise our assumptions about the parts' properties.

2) We can revise our assumptions about the laws that pertain to these properties.

3) We can redescribe the macroscopic behaviour to be explained within the limits defined by experimental accuracy.

Failures of Reductive Explanations?

Did I present you with an *apriori*-argument for the existence of reductive explanations?

No, it’s a hypothesis based on past science: Looking for coherence seems to work as a constitutive principle for doing good science.

As in the case of determinism we would only give up our strive for reductive explanations if there is some theoretical counter-pressure to do so.

V. Controversial cases in Physics

Controversial Cases

Objection: There are cases where we have settled with the failure of reduction!

Case I: Quantum entanglement

Case II: Phase transitions

Quantum entanglement and the end of reductionism

Case I: Quantum entanglement: System consisting of two particles with spin ½, e.g. electrons. Electron 1 is represented in Hilbert-space H_1, electron 2 in H_2.

If we take as a basis for H_1 the eigenvectors in the spin z-direction $|\psi^{z\text{-up}}{}_1>$ and $|\psi^{z\text{-down}}{}_1>$ and as a basis for H_2 $|\psi^{z\text{-up}}{}_2>$ and $|\psi^{z\text{-down}}{}_2>$ we find all of the following among the possible states of the compound system in $H_1 \otimes H_2$:

(i) $|\psi^{z\text{-up}}{}_1> \otimes |\psi^{z\text{-down}}{}_2>$

(ii) $|\psi^{z\text{-down}}{}_1> \otimes |\psi^{z\text{-up}}{}_2>$

(iii) $1/\sqrt{2}\, |\psi^{z\text{-up}}{}_1> \otimes |\psi^{z\text{-down}}{}_2> - 1/\sqrt{2}\, |\psi^{z\text{-down}}{}_1> \otimes |\psi^{z\text{-up}}{}_2>$

(iii) cannot be written as a simple tensor product of vectors of H_1 and H_2. It can only be written as a superposition of such tensor-products.

Quantum entanglement and the end of reductionism

→ The fact that the compound is in a determinate state cannot be explained in terms of determinate states the constituents occupy. The parts on their own have no determinate quantum states.

→ "… reductionism is dead. For the total physical state of the joint system cannot be regarded as a consequence of the states of its (spatially separated) parts, where the states of the parts can be specified without reference to the whole." (Maudlin 1998, 54)

Quantum entanglement and the end of reductionism

Reductive Project 1: give an account of how the behaviour of the compound is tied to the behaviour of the parts in terms of general laws. (part-whole-coherence)

Reductive Project 2: defend a particular ontological view according to which "the world is a vast mosaic of local matters of particular fact, just one little thing and then another. [...] For short [...] an arrangement of qualities. And that is all….," Such a defence requires that all macro-behaviour needs to be explained in terms of actual and local matters of fact.

→ Quantum-entanglement implies the death of project 2 but not quite the death of project 1.

Quantum entanglement and the end of reductionism

All that has been said is compatible with there being an account of how the parts and the compound fit together:

The compound state is written in terms of states that refer to parts:

$$1/\sqrt{2}\, |\psi^{\text{z-up}}_1> \otimes |\psi^{\text{z-down}}_2> - 1/\sqrt{2}\, |\psi^{\text{z-down}}_1> \otimes |\psi^{\text{z-up}}_2>.$$

The Hilbert-space for the compound is the product of the Hilbert-spaces for the parts.

Quantum entanglement and the end of reductionism

Furthermore, it matters that the parts are exactly two and that they are electrons. It matters that the electrons (if on their own) have spin-states, that they are spin-½-particles. And we have a well-defined prescription (direct product) that tells us how to get from the Hilbert-spaces of the parts to the Hilbert-space of the compound.

→ Even if the compound is in a superposition state and the parts do not have determinate states of their own we still have an account of how parts and wholes are related.

Quantum entanglement and the end of reductionism

This is probably not what one expected a part-whole-explanation to look like.

Maybe it is no longer an *explanation*, if an explanation is only allowed to refer to actual states, actual properties etc.

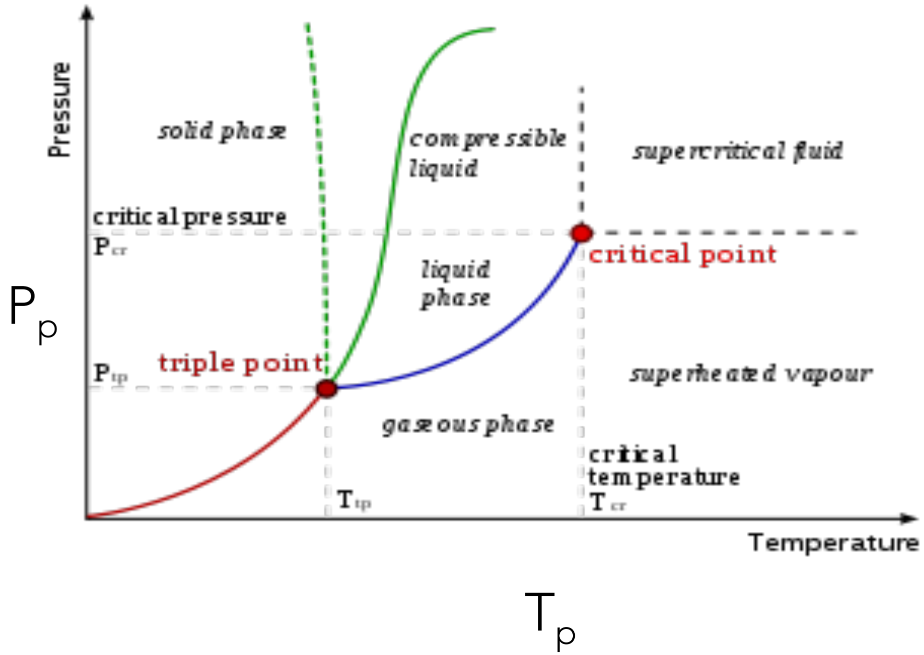
But:

We do have a determinate and explicit account of how the states of the compound are related to ‘information’ about the parts and laws of composition in terms of general laws.

Compared to traditional part-whole-explanations the requirements on what goes into the explanans have been loosened, but the aim of part-whole coherence has not been given up.

→ Project 1 is not entirely dead.

Phase-transitions



Three things need to be distinguished:

- The actual behaviour of the liquid/gas at P_p/T_p
- thermodynamic description of what is going on at P_p/T_p
- statistical mechanical description of what is going on at P_p/T_p

Phase-transitions

Some things that are uncontroversial:

With regard to the real physical system:

There are abrupt changes in behaviour (phases).

At critical points of certain kinds of systems one observes “universal behaviour”

Thermodynamically, phase transitions and critical phenomena are associated with non-analyticities in a system’s thermodynamic functions. Given certain uncontroversial assumptions, such non-analyticities cannot occur in *finite* systems as described by statistical mechanics.

Statistical mechanics can describe phase transitions only in *infinite* particle systems.

Phase-transitions

Characterisation of the problem:

The mere fact that the SM description can account for TD-phase transitions in the thermodynamic limit (i.e. for infinite systems) does not suffice for the reductive projects we are interested in.

Why not?:

a) part whole coherence: We want to understand the behaviour of the compound in terms of its parts. Real systems have only finitely many parts. If it remains unclear how the idealized SM description for the infinite system pertains to the finite system, we do not have an account how the behaviour of the parts (characterised in terms of SM) and that of the compound (characterised in terms of TD) fit together.

b) vertical coherence: We can only understand the successes of TD and why it is legitimate to continue to use TD if the idealized SM description for the infinite system pertains to the finite system too.

Phase-transitions

However: An appeal to infinite idealization need not be a problem:

Illustration:

Not every change in the states of the constituents leads to a change on the macro-level of a compound system. To some extent the macro-behaviour is independent of what is going on at the micro-level. Is there an explanation for the fact that the micro-level "converges on stable macro-level properties?

Basic idea: The central limit theorem implies that fluctuations of macroscopic quantities will in the limit of large N typically decrease as $1/\sqrt{N}$ with system size N. There is a generalization to interacting, thus correlated or dependent random variables.

What is relevant here: stability (i.e. absence of fluctuations) only for infinite systems.

Phase-transitions

Illustration: A magnetic model-system

Ising ferro-magnet on a 3D cubic lattice. Macroscopic magnetic properties in such a system appear as average over microscopic magnetic moments attached to 'elementary magnets' called spins, each of them capable of two opposite orientations in space. These orientations can be thought of as parallel or anti-parallel to one of the crystalline axes.

$S_N (s(t)) = 1/N \; \Sigma_{i}{=}1 \ldots N \; s_i(t)$ (1)

S_N: macro property

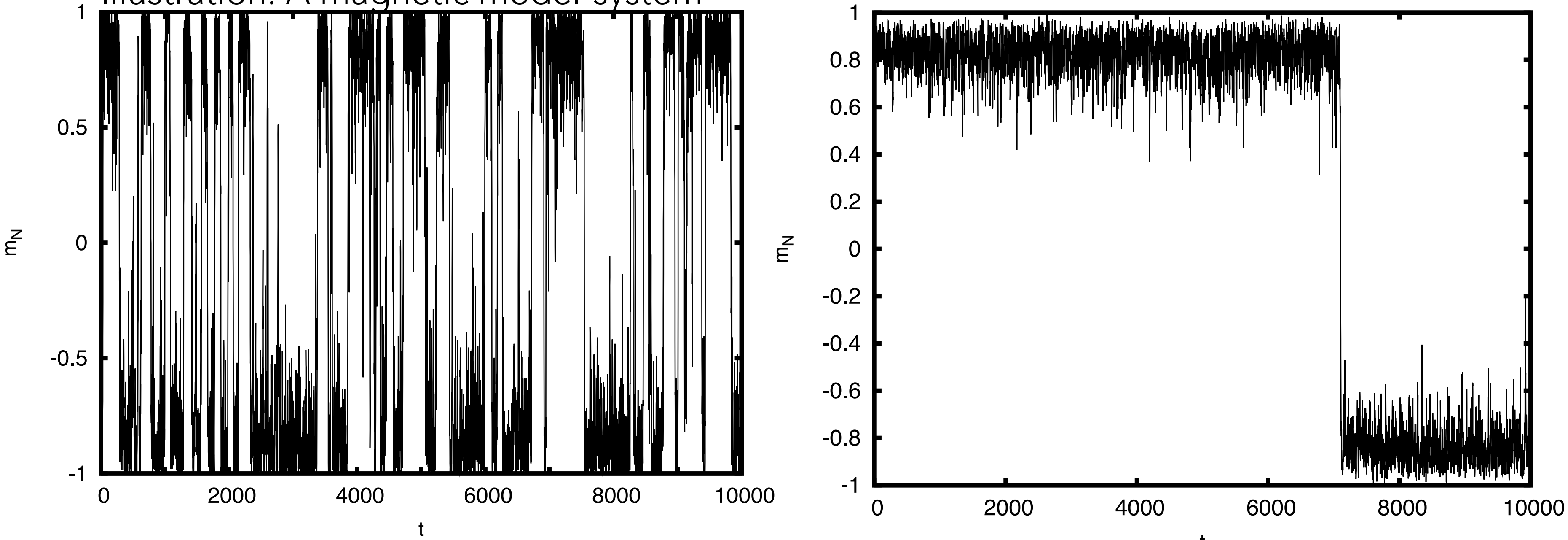
s_i: micro properties

In the following simulation some kind of local interaction is assumed:

$h_i(t) = \Sigma_{j}{=}1 \ldots N \; J_{ij} \; s_j(t)$, which is generated by the other spins.

Phase-transitions

Illustration: A magnetic model-system



Magnetization of a system consisting of $N = 3^3$ spins (left) and $N = 5^3$ spins (right)

Y-axis: $S_N(t)$

X-axis: time-steps

Fluctuations of the magnetization declines with increasing number of spins.

Essential: With increasing system size the system approaches stability smoothly.

(Hüttemann, Kuehn, Terzidis, forthcoming)

Phase-transitions

With increasing system size the system approaches stability smoothly.

→ Appeal to infinity is eliminable.

Why is the appeal eliminable?

We can understand the observed behaviour, i.e. the absence of fluctuations during periods shorter than the age of the universe, in terms of finite systems.

→ Appeal to infinity makes the calculations easier but is not essential.

Phase-transitions

Question:

In the case of phase transitions: is the appeal to infinity eliminable too?

(At least) two issues:

A) Is the appeal to infinity eliminable in the case of the description of phase transitions etc. of singular systems?

B) Is the appeal to infinity eliminable in the explanation of universality?

(I will focus on A)

Phase-transitions

One view: The limit ($\lim_{N\to\infty} T_N = T_O$) does not obtain in the case of phase transitions because there is a singularity at the limit. If the limit is *singular* "the exact solution for $\varepsilon = 0$ is *fundamentally different in character* from the 'neighboring' solutions obtained in the limit $\varepsilon \to 0$." (Bender and Orszag, 1978, p. 324, quoted by Batterman)

The 'neighboring' (i.e. finite-system) solutions cannot explain phase transitions. Therefore the appeal to infinity is ineliminable. Thus:

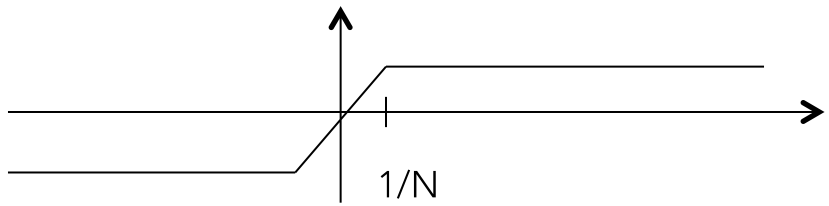
→ (i) We cannot explain the success of TD on the basis of SM because we cannot explain the *existence* of phase-transitions on the basis of SM.

→ (ii) We cannot explain the behaviour of the compound system in terms of the behaviour of the parts as characterised by SM, because the compound displays a phase-transition, which is not defined in SM.

Phase-transitions

(i) We cannot explain the success of TD on the basis of SM because we cannot explain the existence of phase-transitions on the basis of SM

Rejoinder: Start with



$g_N(x) = 1$ for $x > 1/N$

$g_N(x) = -1$ for $x < -1/N$

$g_N(x)$ is continuous for finite N,

discontinuous for $N=\infty$

$f_N(x) = 0$ provided $g_N(x)$ is continuous at x;

$f_N(x) = 1$, provided $g_N(x)$ is discontinuous at x

$f_N(x) = 0$ for all finite N but in the limit $N=\infty$ it equals 1.

If a theory is written in terms of *f* it cannot be seen how *f* for the infinite limit can be approximated by finite cases. But that is an artifact of the presentation. (Butterfield 2011; Menon & Callendar 2012).

Phase-transitions

(ii) We cannot explain the behaviour of the compound system in terms of the behaviour of the parts as characterised by SM, because the compound displays a phase-transition, which is not defined in SM.

Rejoinder:

Even in the absence of an answer to (i), one may argue: What we actually observe in finite systems and describe by using the expression "phase-transition" need not be described as a non-analyticity. Our observations cannot discriminate between a non-analytic function and a very similar (rounded off) analytic function.

So, even though we cannot for finite systems reductively explain (in terms of SM) the occurrence of phase-transitions *as defined in TD* we can reductively explain *the observed behaviour*. That suffices for part-whole-coherence.

Phase-transitions

And wouldn't that give us vertical coherence too?

Suppose we have a part-whole explanation in terms of SM for all finite systems provided we can – within experimental accuracy – revise the TD-characterisation of the compound system (i.e. replace description of the observed behaviour in terms of "TD-phase-transition" by sth. that does not require going to the TD-limit)

If this is possible we have an explanation of why the old theory was empirically successful within a certain range and why we can continue to use it within certain confines. (vertical coherence)

→ So even in the controversial cases it seems we are quite successful of achieving coherence (vertical or part-whole).