Metamorphoses of a photon sphere

Abstract

There are circular planar null geodesics at $r = 3/4 \, M$ around a Schwarzschild black hole of mass $M$. These geodesics form a photon sphere. Null geodesics of the Schwarzschild space-time which do not form the photon sphere are either escape to null infinity or get captured by the black hole. Thus, from the dynamical point of view, the photon sphere represents a smooth basin boundary that separates the basins of escape and capture of the dynamical system governing the null geodesics. Here we consider a Schwarzschild black hole distorted by an external, static, and axisymmetric quadrupolar gravitational field defined by a quadrupole moment $q$. We study null geodesics around such a black hole and show that the photon sphere does not survive the distortion. For $q \leq -0.017001$ it transforms into a fractal basin boundary that indicates chaotic behavior of the null geodesics. We calculate the box-counting fractal dimension of the basin boundary and the related uncertainty exponent, which depend on the value of the quadrupole moment.