Quantum Spacetime, from a Practitioner's Point of View

Bremen, 5 Oct 2012

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Introduction

Talking about "Models of Gravity", what we are still lacking is a *completion* of the theory of General Relativity (or perhaps of alternative, classical theories) on short distances. This is what one usually means by *Quantum Gravity*. In particular, answering the question "What does spacetime look like at the very shortest scales?" requires an understanding of the *quantum* dynamics of gravity.

I will discuss today a specific quantum gravity theory and what it has to say about the quantum origins of spacetime. This will take the form of an exploration of the *dynamics* of a statistical mechanical system of quantum spacetime, which (in 4d) is conducted largely via numerical "experiments".

Because of its small characteristic length scale, $\ell_{\rm Pl} = \sqrt{G_{\rm N}\,\hbar/c^3} \approx 10^{-35} m$, there is little in terms of phenomenology to guide our search for *Quantum Gravity*. Nevertheless, in the specific theory I will discuss, one can use powerful nonperturbative methods to extract "predictions", and uncover generic properties of the dynamics of Planckian geometric excitations.

Plan for this talk

- give a working definition of "quantum spacetime"
- present a lightning review of CDT ("Causal Dynamical Triangulations") quantum gravity and explain motivation, ingredients, results and new developments

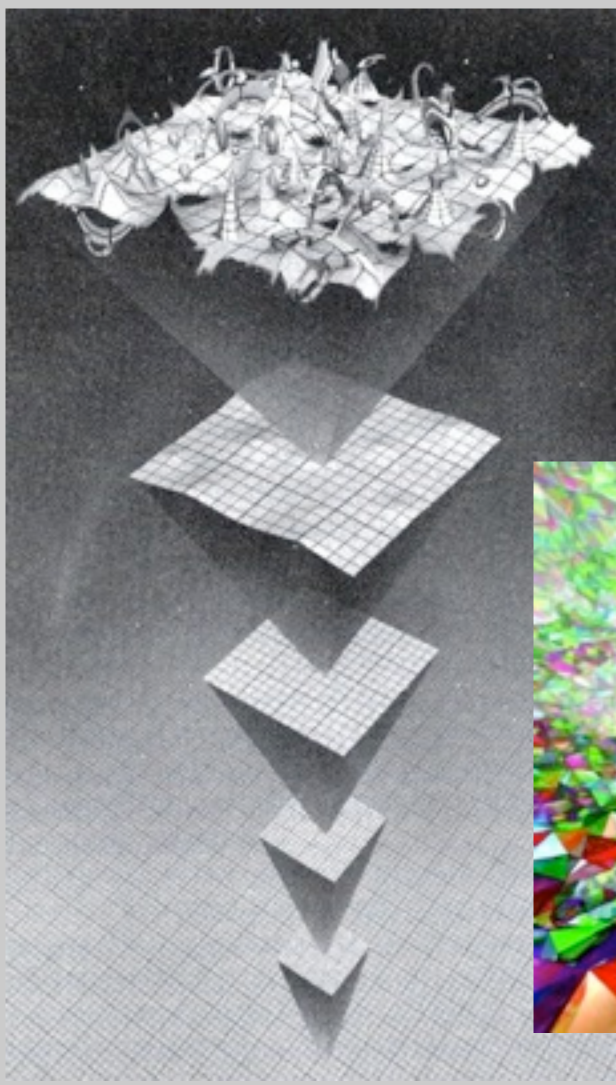
By way of doing this,

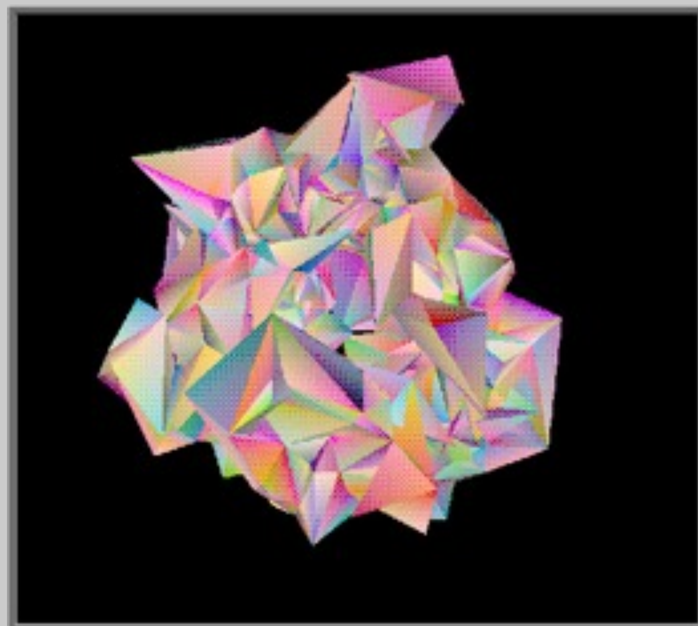
- give you an idea of the essence and prospects of this quantum gravity theory
- guide your expectations (w.r.t. properties of quantum spacetime)
- make you think about possible implications for your own approach
- ask and try to answer some fundamental questions about methodology and the physics of quantum gravity

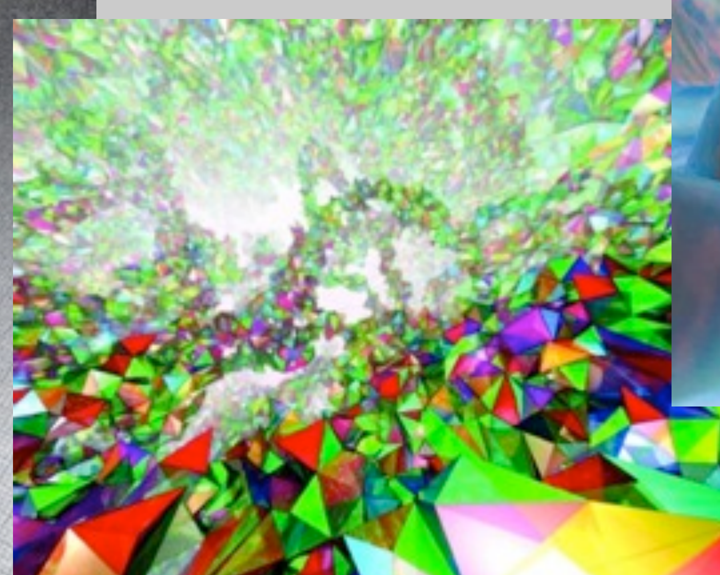
What *is* quantum spacetime?

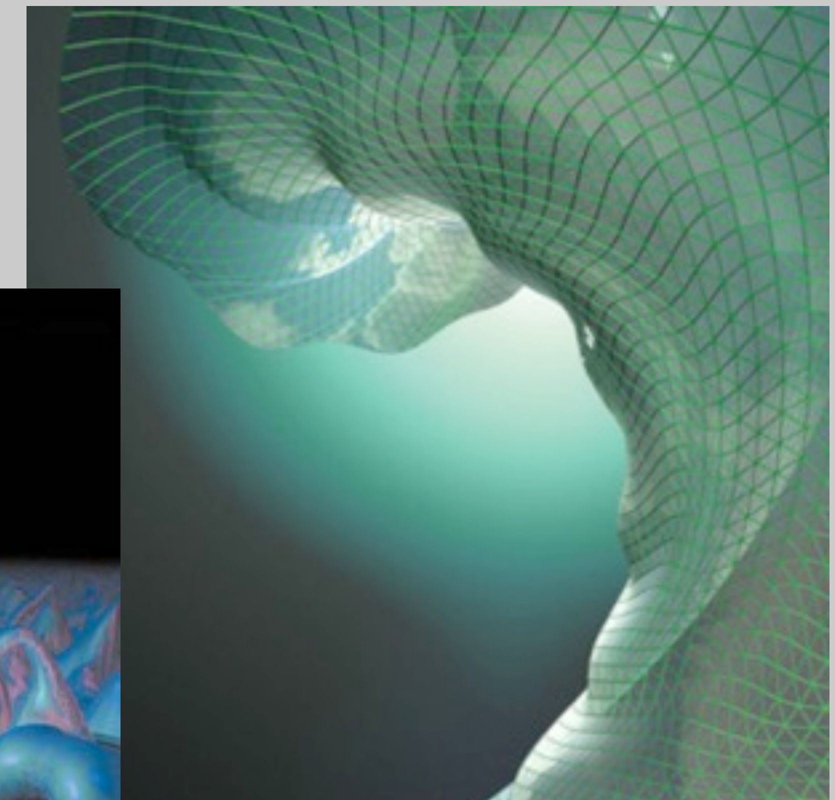
A "spacetime" with quantum properties $\sim \ell_{\mathrm{Pl}}$, which in a suitable macroscopic limit can be approximated by a classical curved spacetime $(M^{(4)}, g_{\mu\nu}(x))$, mod $\mathit{Diff}\,(M)$.

artistic impressions of "quantum foam":









Which classical geometric features continue to be meaningful at the Planck scale?

How to get to a quantum spacetime?

Input —implementation→ **Output**

- microscopic ingredients/ "degrees of freedom"
- kinematical principles/ symmetries/algebraic structures
- a given classical spacetime (*but* ↯ perturb. nonrenormaliz.!)

} *"background"*

- dynamical principle

- *a* quantum spacetime?
- a fundamental theory of quantum gravity, valid on all scales?

it is highly nontrivial to produce an "output" = results

we *hope* that the outcome is as robust/unique as possible

In a dynamical framework, how ambitious can we be? - *Some* background structure is always needed (e.g. in path integral).

(i) Which features are dynamically determined, without being put in? - "geometry", "topology", "dimensionality", ...?

(ii) Which aspects of classical geometry are merely "emergent" macroscopically? - "time", "causality", ...?

The most fruitful and concrete ideas on how to make progress are coming from a "new minimalism" in quantum gravity.

→ no strings, loops, branes, extra dimensions, new symmetries, landscapes, multiverses, ...

A working hypothesis which is becoming ever more popular:

The framework of standard quantum field theory is sufficient to construct and understand quantum gravity as a fundamental theory, if one properly takes into account the dynamical, causal and nonperturbative nature of spacetime.

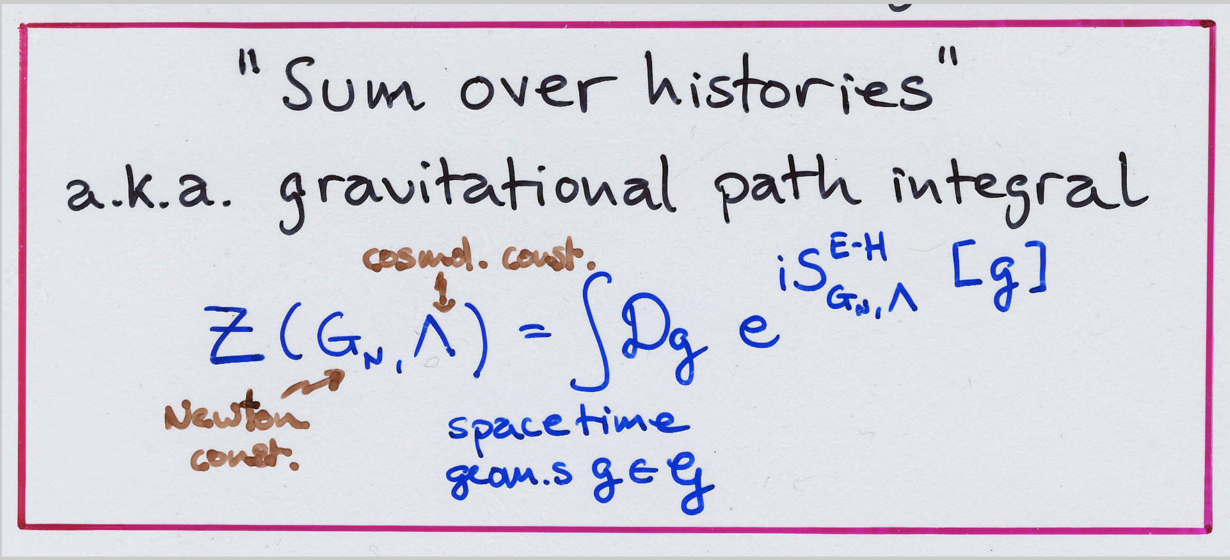
Significant support for this comes from a (relatively) new candidate theory,

"Quantum Gravity from Causal Dynamical Triangulation (CDT)",

which is a background-independent and nonperturbative approach that has already passed nontrivial tests and has produced genuinely new, quantitative(!) results. Its ambition is to *explain* gravity, (quantum) spacetime and - by implication - cosmology from microscopic first principles.

Key points of the CDT approach:

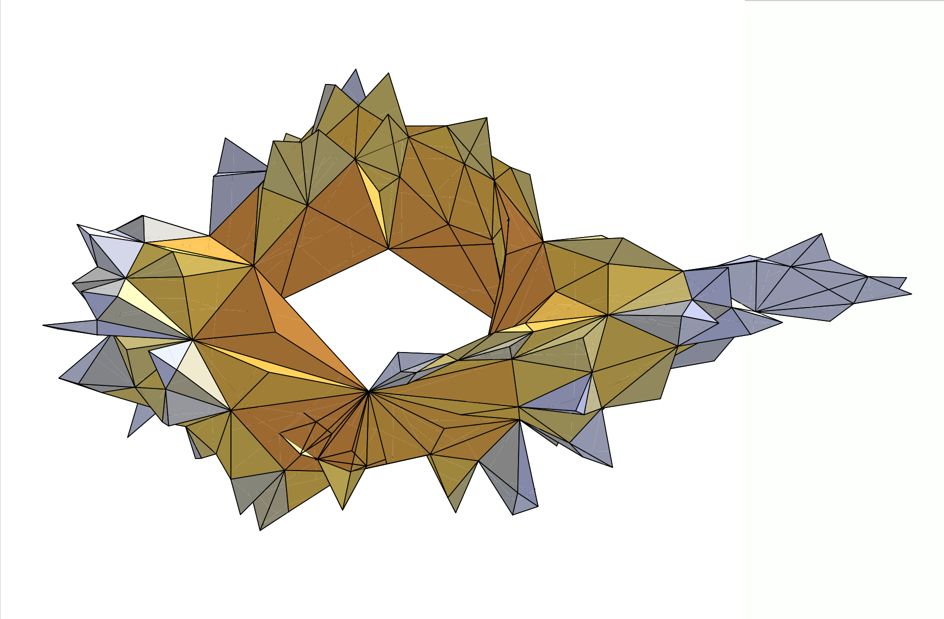
- Few ingredients/priors:
  - quantum superposition principle
  - locality and causality
  - notion of (proper) time
  - Wick rotation
  - standard tools of quantum field theory
- Few free parameters (Λ, G_N, Δ)
- Robustness of construction; universality
- at an intermediate stage, one approximates curved spacetimes by triangulations
- Availability of nonperturbative computational tools to extract quantitative results



making sense of the SOH

I will give a short description of the set-up, then look at some of the results produced:

- nontrivial phase structure
- "emergence" of spacetime
- a totally new view on "dimensionality"

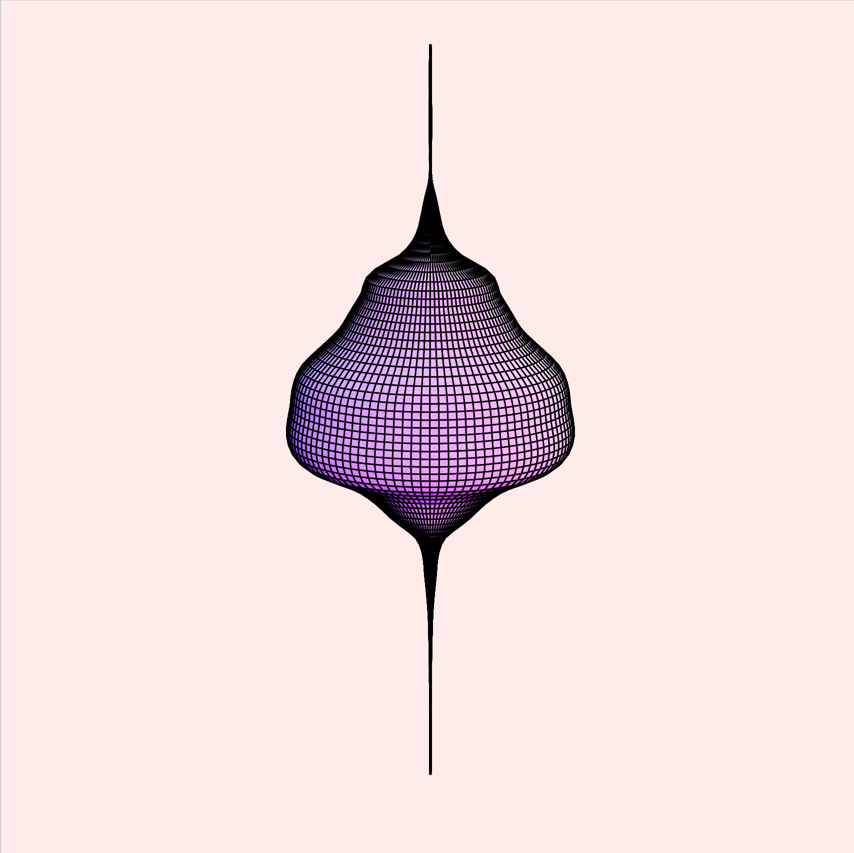


triangulated torus

Quantum Gravity from Causal Dynamical Triangulation (QG from CDT)✶

CDT is a no-frills nonperturbative implementation of the gravitational path integral, much in the spirit of lattice quantum field theory, but based on *dynamical* lattices, reflecting the dynamical nature of spacetime geometry.

A key result that puts QG from CDT on the map as a possible quantum theory of gravity is the fact that it can generate dynamically a background geometry with semiclassical properties from pure quantum excitations, in an a priori background-independent formulation. (C)DT has also given us crucial *new* insights into nonperturbative dynamics and pitfalls.

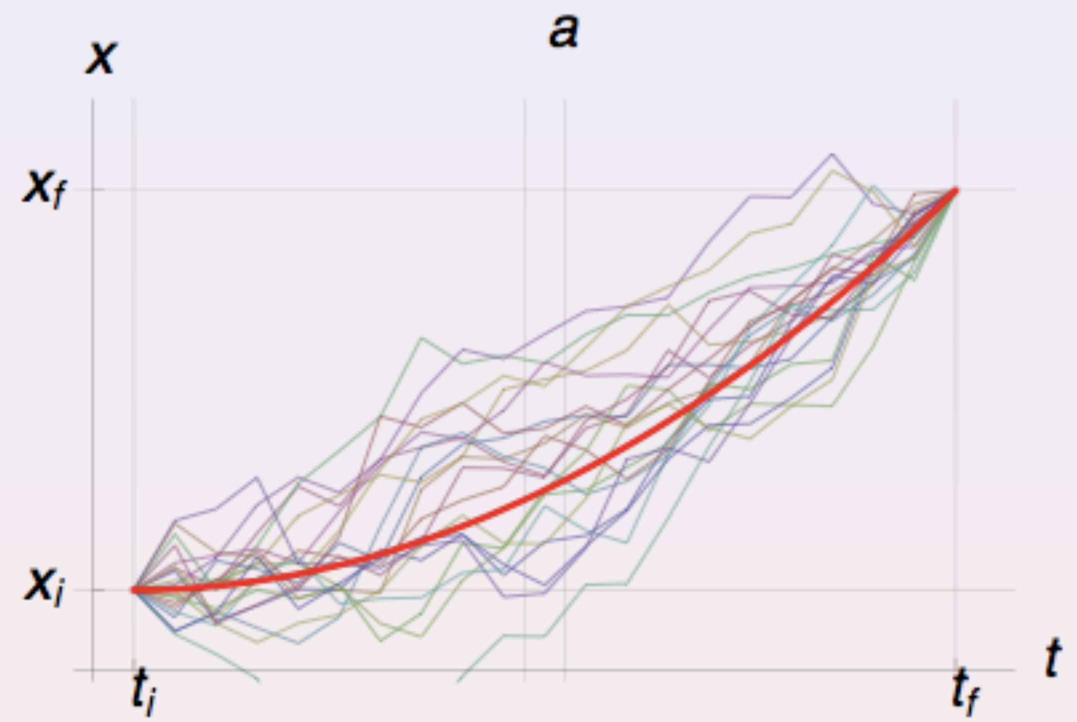


(PRL 93 (2004) 131301, PRD 72 (2005) 064014, PLB 607 (2005) 205)

✶ the results presented are based on joint work with J. Ambjørn, J. Jurkiewicz, T. Budd, A. Görlich and S. Jordan

Basic tool: the good old path integral

Textbook example: the nonrelativistic particle (h.o.) in one dimension



$$x(t) = \langle x(t) \rangle + y(t)$$
$$\langle |y| \rangle \propto \sqrt{\hbar / m\omega}$$

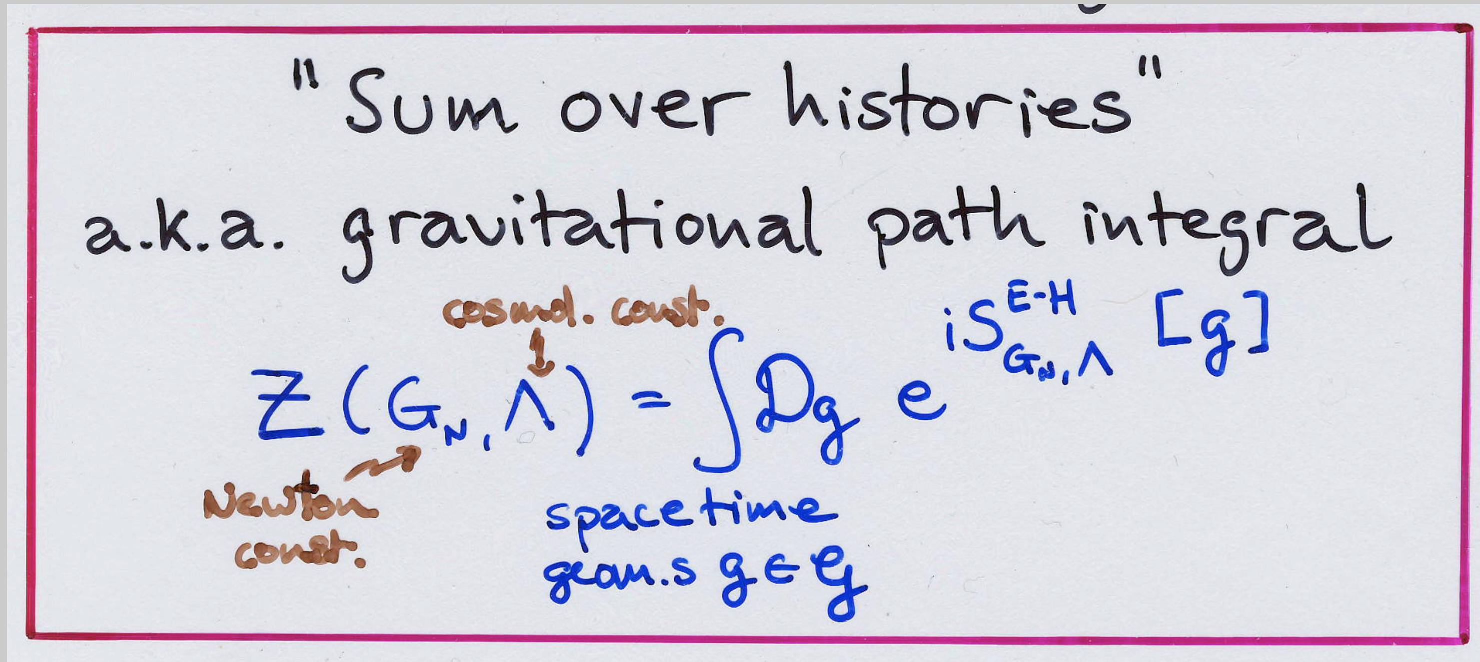
In QG we want $\langle x(t) \rangle$

$$\langle |y| \rangle \propto \sqrt{\hbar G}$$

Quantum superposition principle: the transition amplitude from $x_i(t_i)$ to $x_f(t_f)$ is given as a weighted sum over amplitudes exp iS[x(t)] of all possible trajectories, where S[x(t)] is the classical action of the path.

(here, time is discretized in steps of length *a*, and the trajectories are piecewise linear)

The same superposition principle, applied to gravity



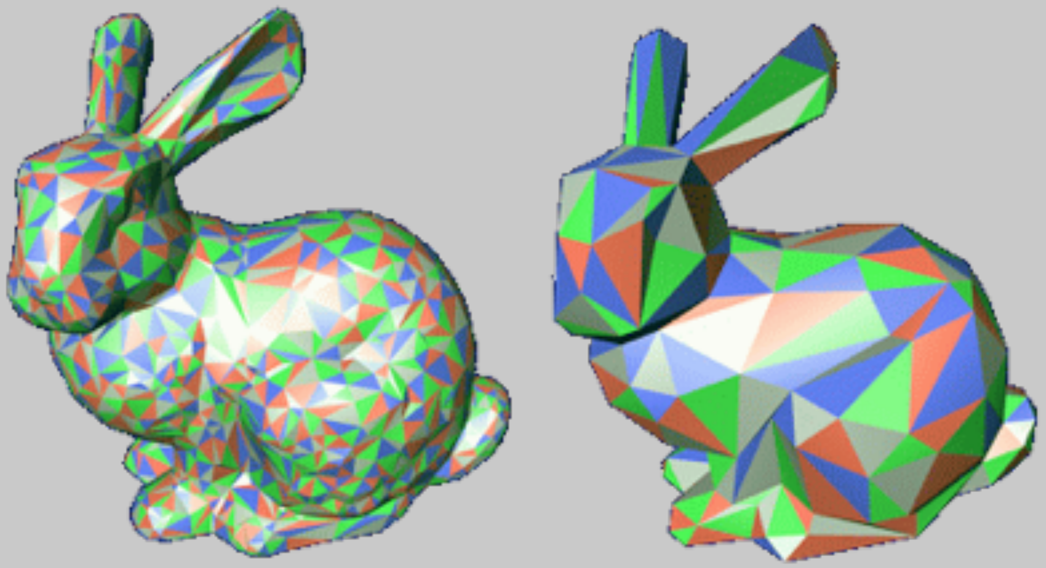
Each "path" is now a four-dimensional, curved spacetime geometry g, which can be thought of as a three-dimensional, spatial geometry developing in time. The weight associated with each g is given by the corresponding Einstein-Hilbert action $S^{EH}[g]$,

$$S^{\mathrm{EH}} = \frac{1}{G_{\mathrm{N}}} \int d^4x \sqrt{-\det g}\,(R[g, \partial g, \partial^2 g] - 2\Lambda)$$

How can we make $Z(G_N,\Lambda)$ into a meaningful, well-defined quantity?
How do we get a handle on the *space of all spacetimes*?

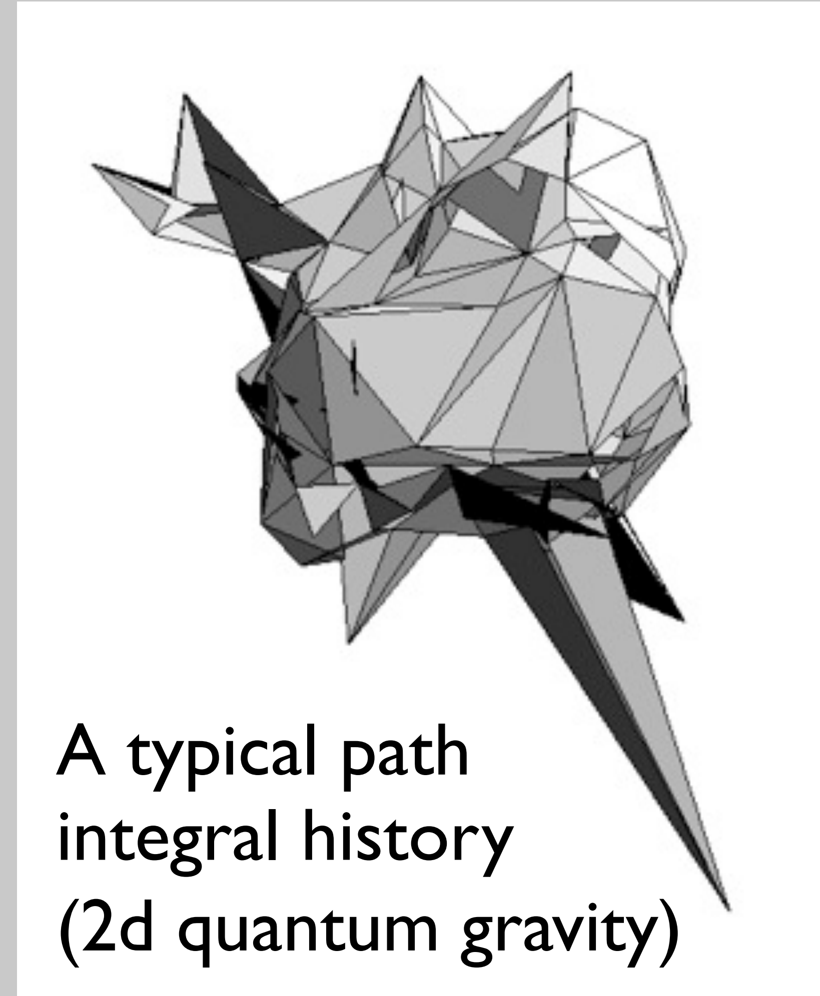
"General Relativity without Coordinates" (Regge 1961)

A key input in dynamical triangulations



approximating *classical* curved surfaces through triangulation

triangulation = regularization

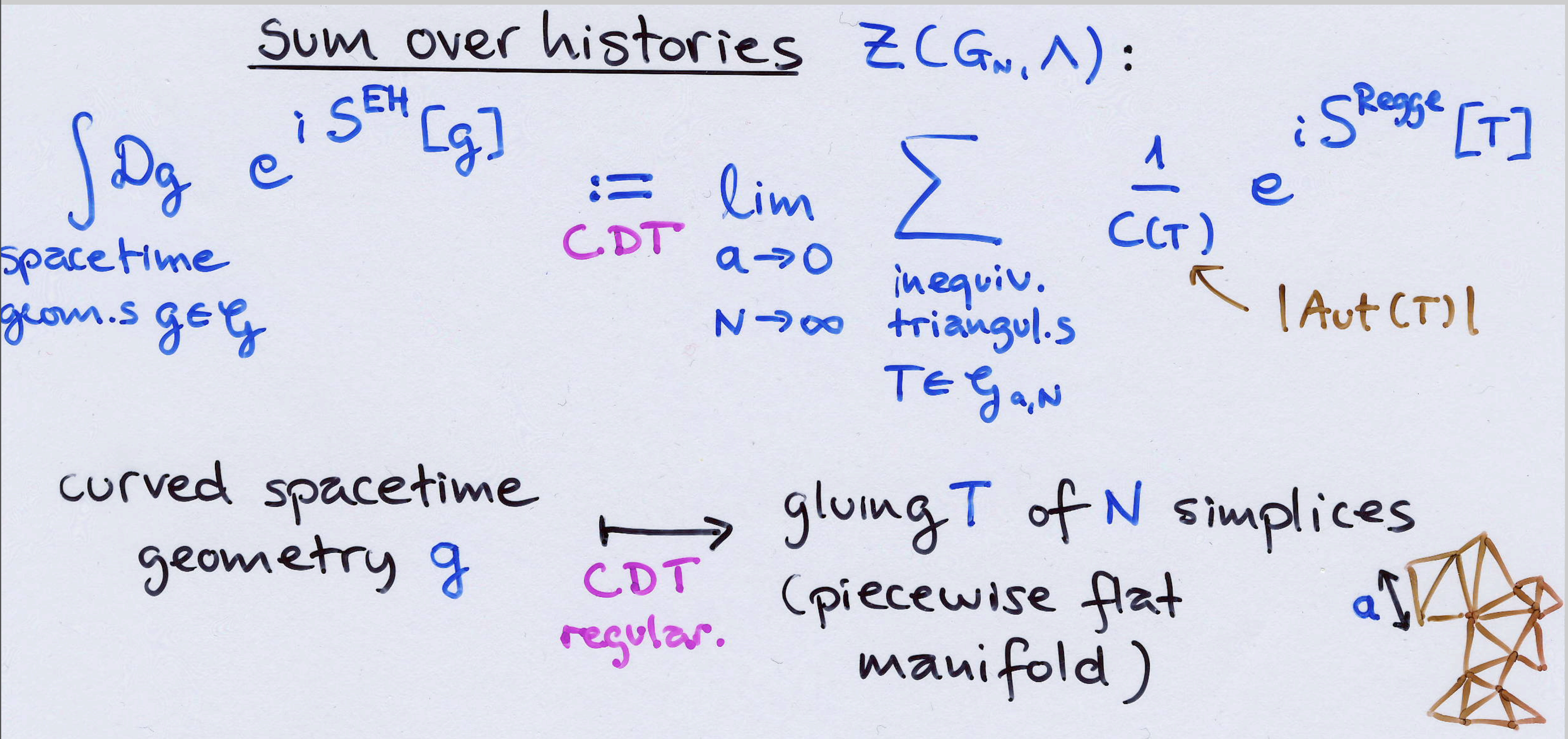


A typical path integral history (2d quantum gravity)

Quantum Theory: approximating the space of all curved geometries by a space of triangulations - one needs to integrate over this space(*)!

(*) *by Monte Carlo simulations* (for CDT models in d=2, 3 have also exact stat. mech. solutions methods, see e.g. D. Benedetti, F. Zamponi, R.L., PRD 76 (2007) 104022; in d=2, the problem is exactly soluble - nontrivial propagator, Hamiltonian, ..., work by J. Ambjørn, R.L., P. di Francesco, E. Guitter, C. Kristjansen, B. Durhuus, ...)

Regularizing the path integral via CDT

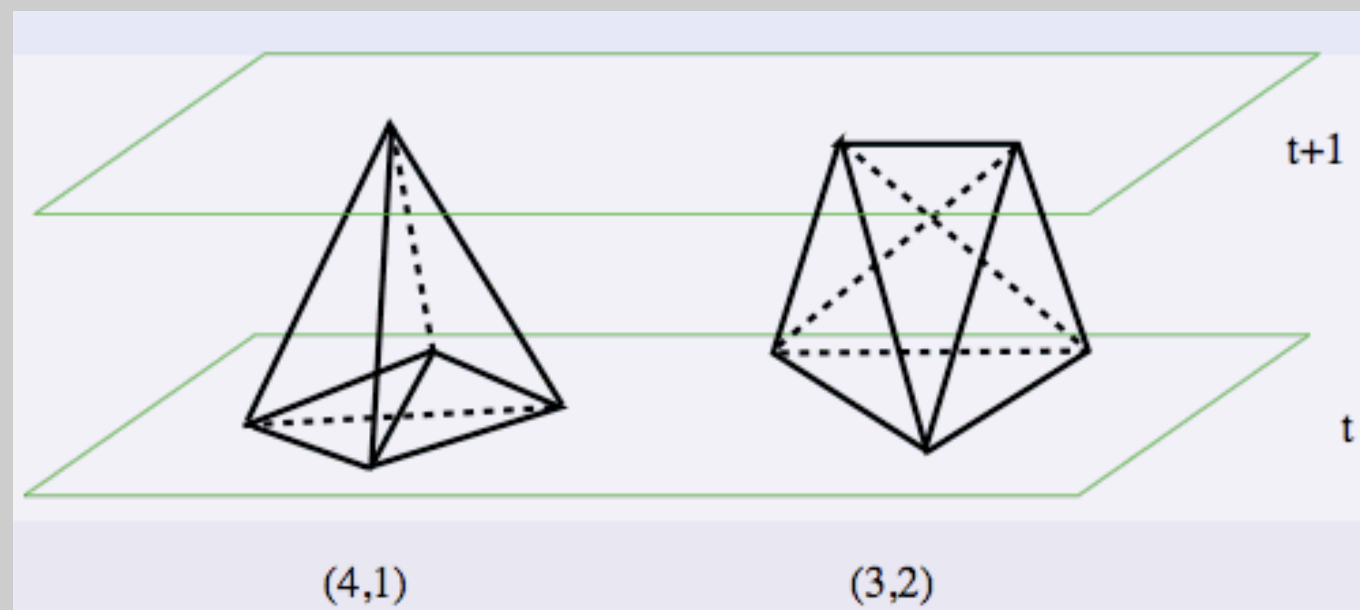


'democratic', regularized sum over piecewise flat spacetimes, *doesn't need coordinates* (Regge); continuum limit required to obtain universal results independent of the regularization

Elementary four-simplex, building block for a causal dynamical triangulation:

a ~ edge length; diffeomorphism-invariant UV regulator

Causality is essential! This does not work in Euclidean signature - get only branched polymers (~mid-90s).



CDT's proper-time slicing -
time and space are not equivalent

Wick rotation and analogy with statistical mechanics

- each regularized Lorentzian (-+++) geometry T allows for a rotation to a unique regularized Euclidean (++++) geometry T_{eu}, such that the Feynman amplitude of a path is turned into a Boltzmann weight, as in statistical mechanics

$$\mathrm{e}^{iS^{\mathrm{Regge}}(T)} \longrightarrow \mathrm{e}^{-S^{\mathrm{Regge}}_{\mathrm{eu}}(T_{\mathrm{eu}})}$$

- this turns the *complex* quantum amplitude Z into a *real* partition function Z_{eu} and allows us to use powerful numerical methods from statistical mechanics, including Monte Carlo simulations

- a ‘classical trajectory’ is an average over quantum trajectories in the statistical ensemble of trajectories (the Euclideanized ‘sum over histories’)

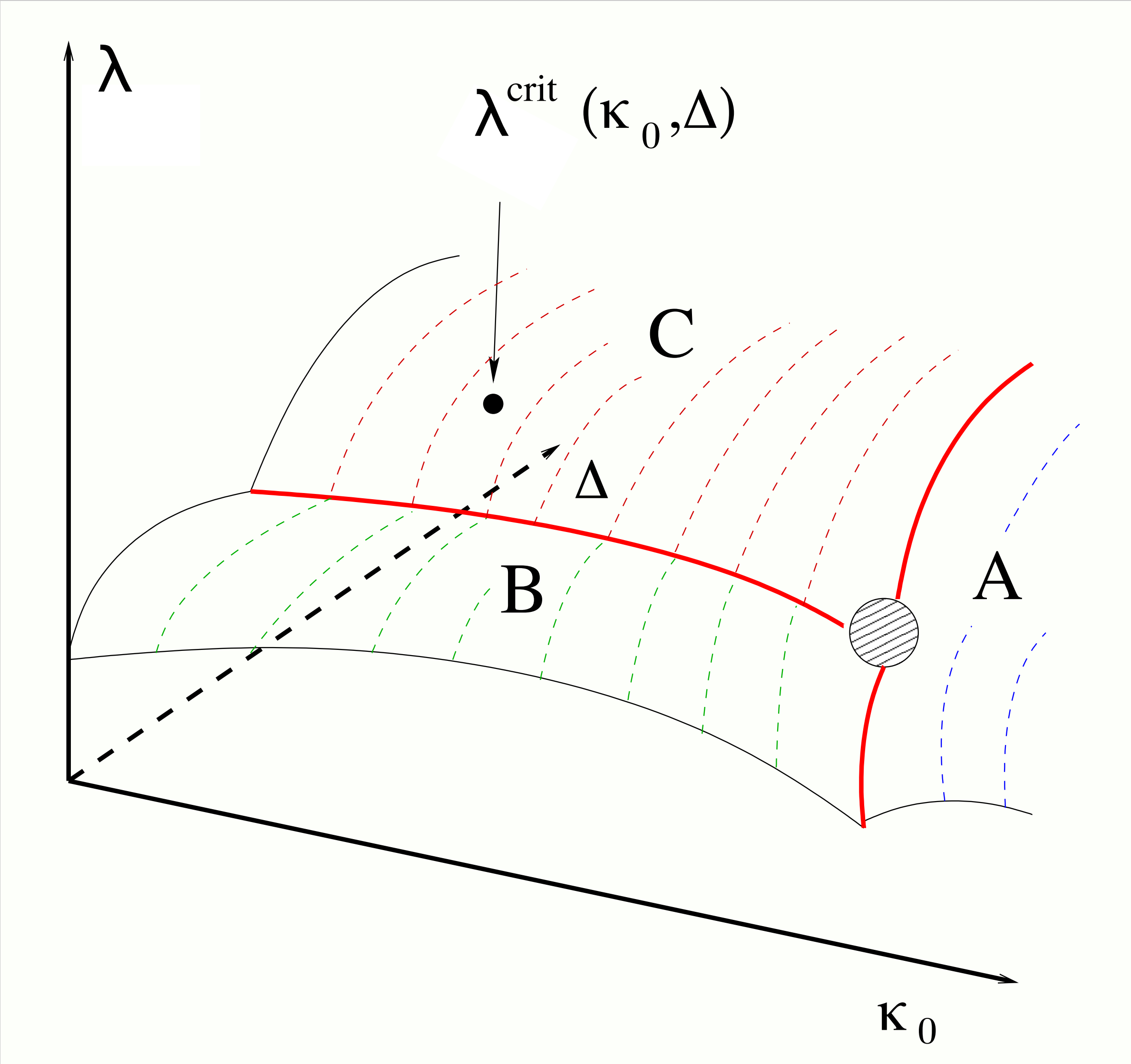
- taking the continuum limit of this regularized theory means studying the critical behaviour of the underlying statistical theory

$$S^{\mathrm{EH}} = \frac{1}{G_N}\int d^4x \sqrt{-\det g}\,(R[g,\partial g,\partial^2 g] - 2\Lambda)$$

- performing an ‘inverse Wick rotation’ on quantities computed in the continuum limit is in general nontrivial

Our toolbox is ready - now for some *results*.

The phase diagram of Causal Dynamical Triangulations



The gravitational action is *simple*:

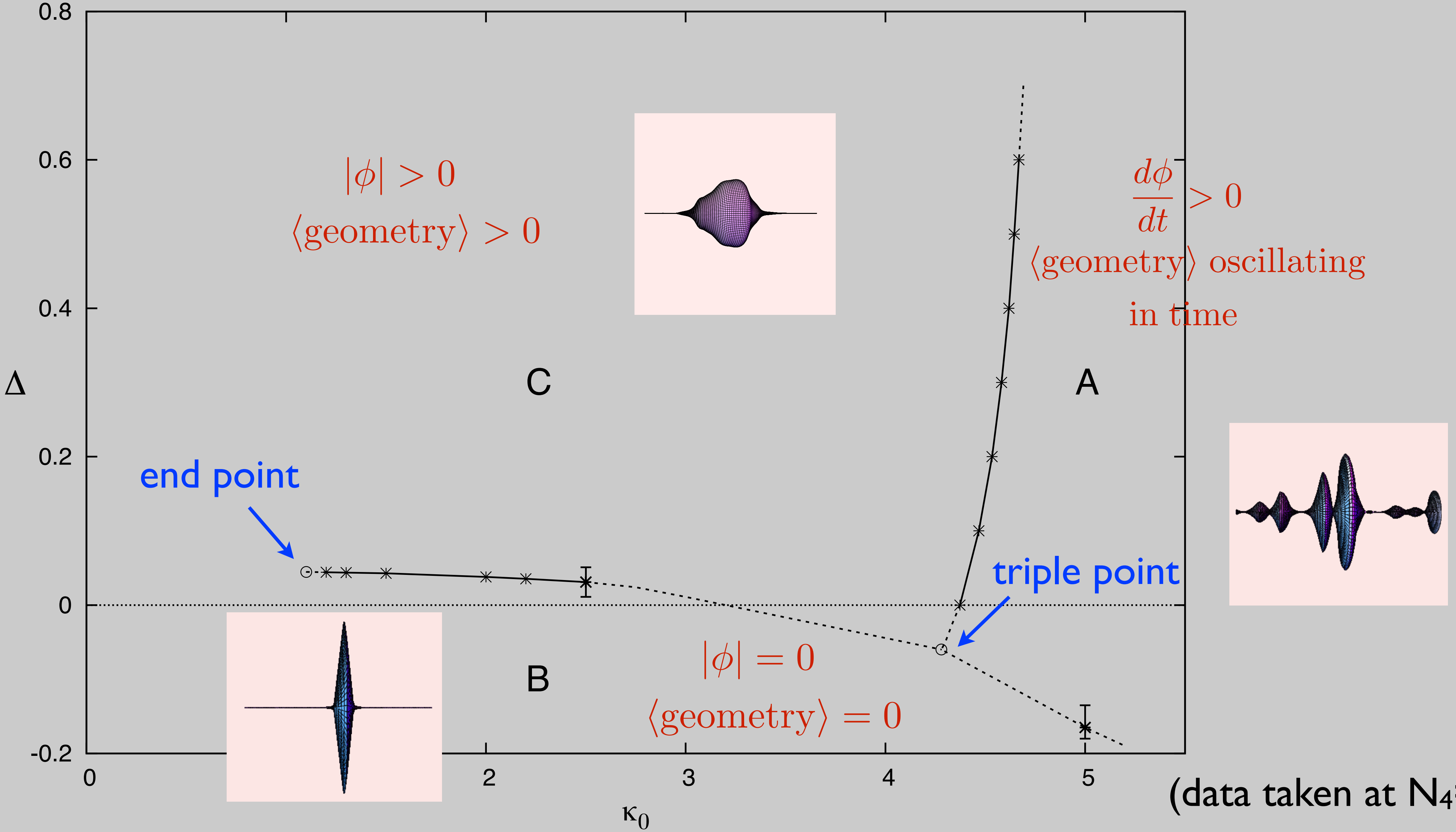
$$S_{eu}^{\text{Regge}} = -\kappa_0 N_2 + N_4(c\kappa_0 + \lambda) + \\ +\Delta(2N_4^{(4,1)} + N_4^{(3,2)})$$

- λ ~ cosmological constant
- κ_0 ~ $1/G_N$ inverse Newton's constant
- Δ ~ relative time/space scaling
- c ~ numerical constant, >0
- N_i ~ # of triangular building blocks of dimension i

The partition function is defined for $\lambda > \lambda^{crit}(\kappa_0,\Delta)$;
approaching the critical surface from above = taking the infinite-volume limit.
red lines ~ phase transitions

(J. Ambjørn, J. Jurkiewicz, RL, PRD 72 (2005) 064014;
J. Ambjørn, A. Görlich, S. Jordan, J. Jurkiewicz, RL, PLB 690 (2010) 413)

The phase diagram of CDT in the κ_0-Δ plane



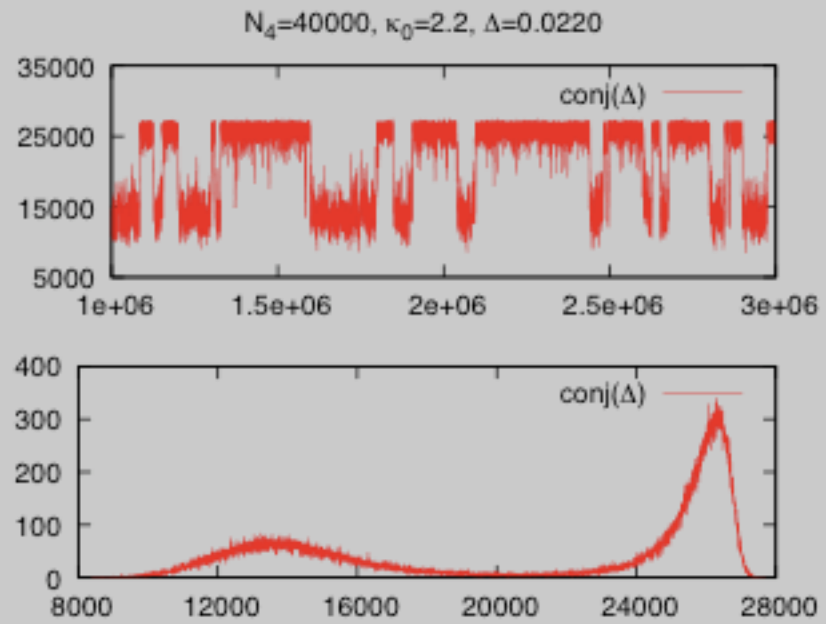
(data taken at N_4=80.000)

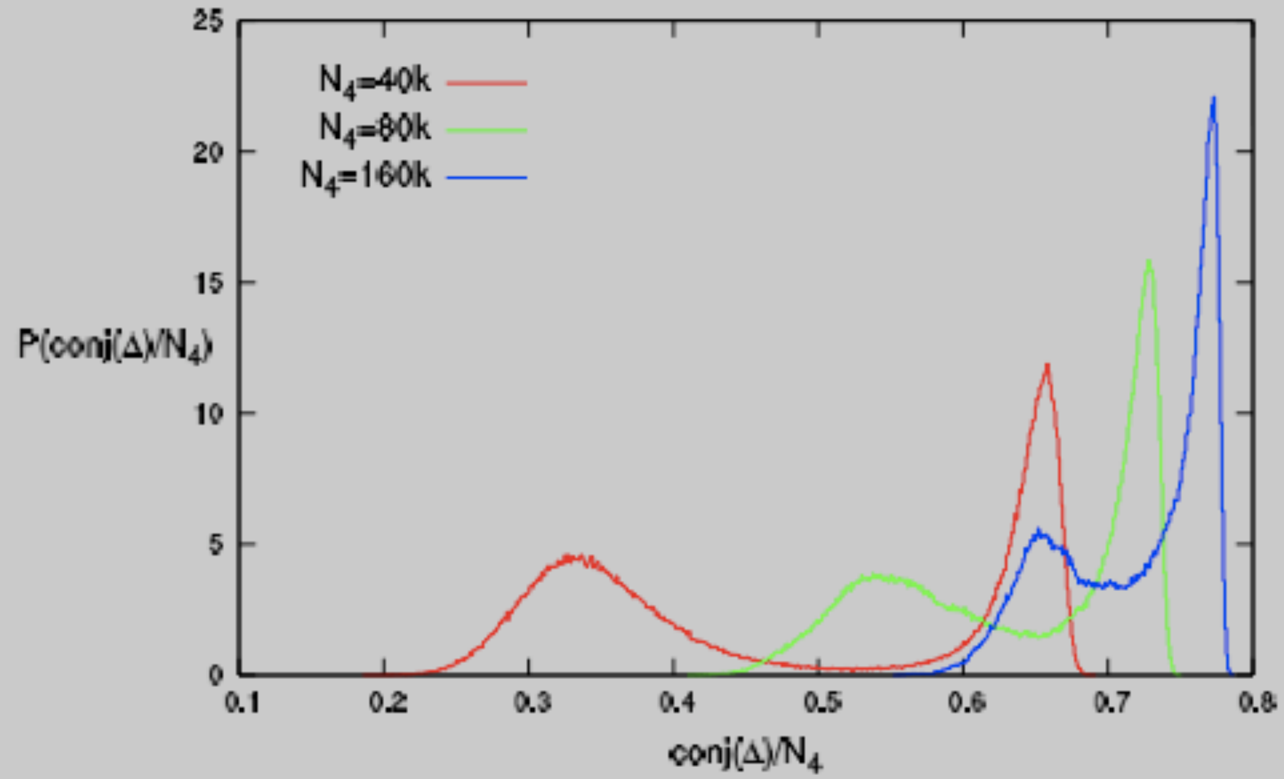
Perhaps surprisingly, the *average* geometry in phases A and B is degenerate and does *not* possess a classical, four-dimensional limit.

Exciting news: evidence that the B-C transition is "second order"! (J. Ambjørn, S. Jordan, J. Jurkiewicz, R.L., PRL 107 (2011) 211303; PRD 85 (2012) 124044)

(note similarity with Lifshitz phase diagram, P. Hořava, Class. Quant. Grav. 28 (2011) 114012)

The evidence:



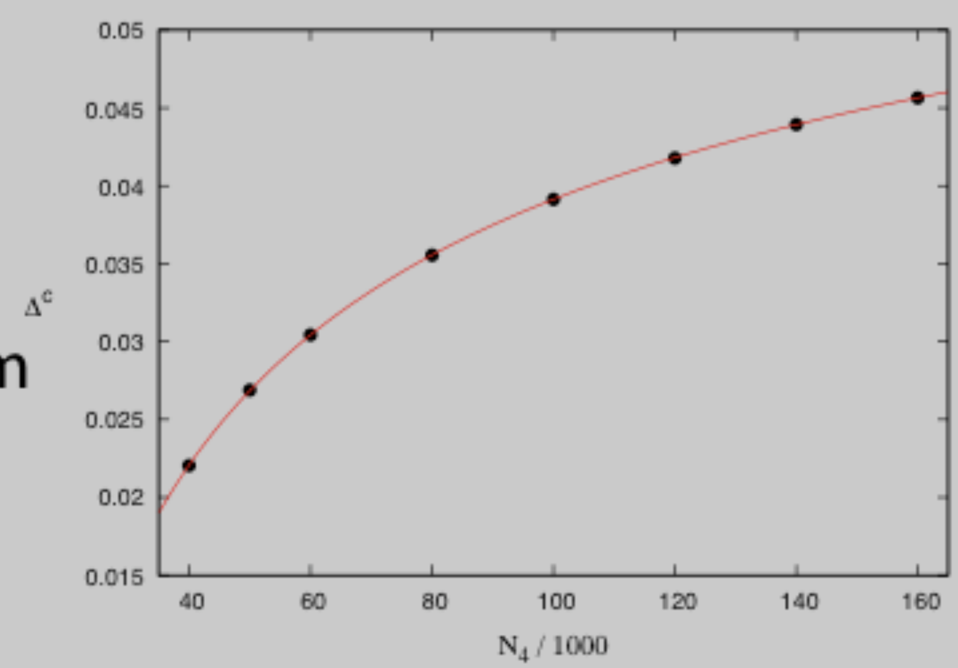


peaks move closer with increasing volume

Monte Carlo time evolution of conj(Δ) at the B-C transition with associated histogram

extracting the shift exponent ν from measuring the location of the maximum of the susceptibility of conj(Δ):

$$\Delta^c(N_4) = \Delta^c(\infty) - C N_4^{-1/\tilde{\nu}}$$



ν=2.51(3)

(should be =1 for a first-order trans.)

The dynamical emergence of spacetime as we know it

CDT is the so far only candidate theory of nonperturbative quantum gravity where a classical extended geometry is generated from nothing but Planck-scale quantum excitations.

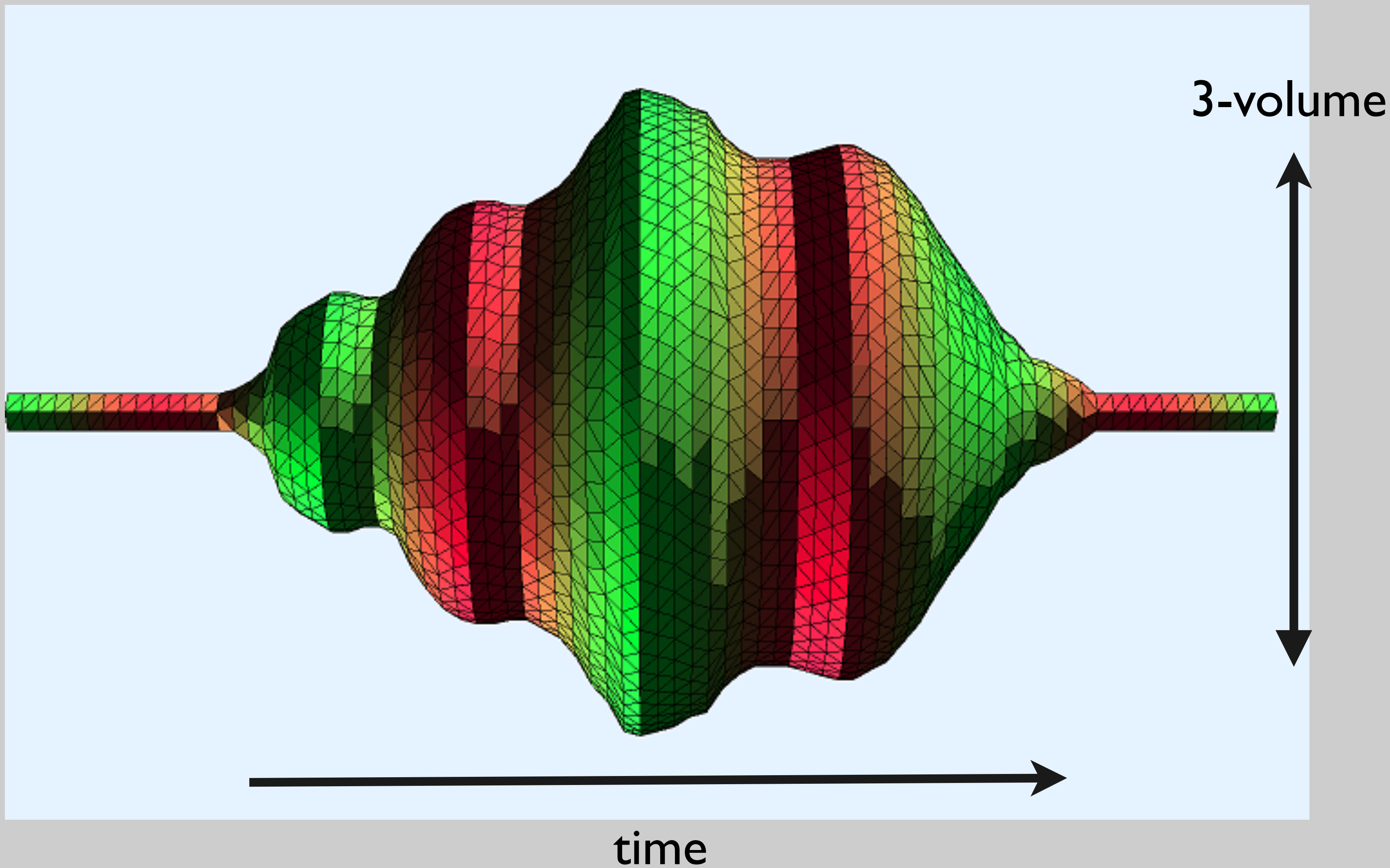
This happens by a nonperturbative, entropic✶ mechanism:

Magically, in phase C, the many microscopic building blocks in the quantum superposition arrange themselves into an extended 4d quantum spacetime whose macroscopic shape is that of a well known cosmology.

Evidence: When, from all the gravitational degrees of freedom present, we monitor only the average spatial three-volume $\langle V_3(t) \rangle$ of the universe as a function of proper time t, we find a characteristic "volume profile".

✶entropy = number of microscopic geometric realizations of a given value of the action

Dynamically generated four-dimensional quantum universe, obtained from a path integral over causal spacetimes (phase C)



This is a Monte Carlo "snapshot" of spacetime shape (a volume profile $V_3(t)$) - we still need to average to obtain its *expectation value* $\langle V_3(t) \rangle$.

Our "self-organized quantum spacetime" has the shape of a de Sitter universe!

(A solution to the classical Einstein equations in the presence of "dark energy" - a.k.a. a cosmological constant Λ.)

Classical de Sitter space has

$$V_3(\tau) = 2\pi^2 \left(c \cosh \frac{\tau}{c}\right)^3 \quad \leftarrow \text{constant},$$

giving rise to an exponentially expanding universe, $V_3 \sim e^{c'\tau}$, for $\tau > 0$.

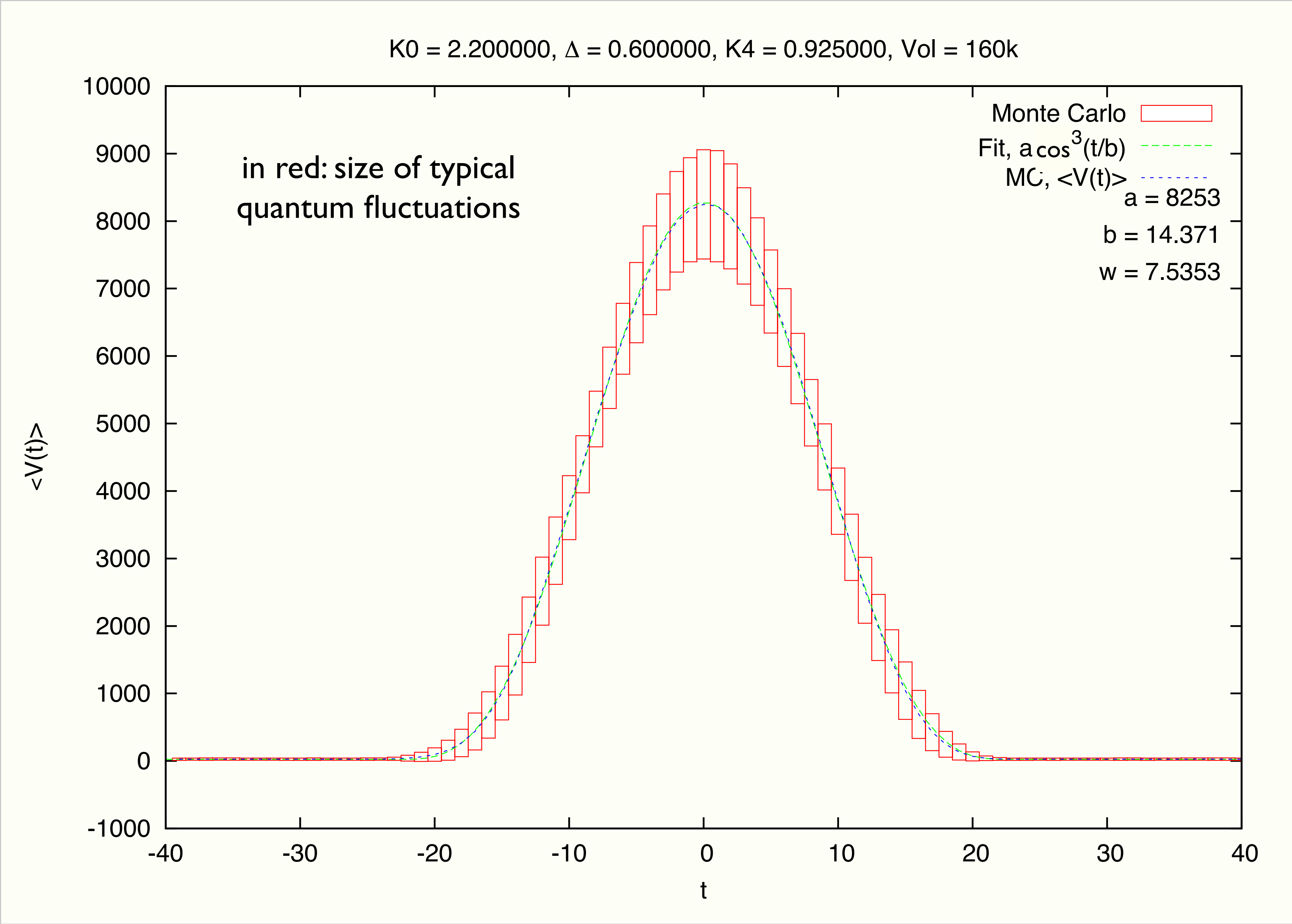
- We measure this for Euclidean time $t = i\tau$

(τ denotes physical proper time; $c \sim \Lambda^{-1/2}$)

→ a very nontrivial test of the classical limit; strong flavour of condensed matter phenomena

(J. Ambjørn, A. Görlich, J. Jurkiewicz, RL, PRL 100 (2008) 091304, PRD 78 (2008) 063544, NPB 849 (2011) 144 (with J. Gizbert-Studnicki, T. Trzesniewski))

What is the quantitative evidence for de Sitter space?



The volume profile $\langle V_3(t)\rangle$, as function of Euclidean proper time t=iτ, perfectly matching that of a Euclidean *de Sitter space*, with scale factor $a(t)^2$ given by

$$ds^2 = dt^2 + a(t)^2 d\Omega_{(3)}^2 = dt^2 + c^2 \cos^2\left(\frac{t}{c}\right) d\Omega_{(3)}^2 \longleftarrow \text{volume el. } S^3$$

(N.B.: we are *not* doing quantum cosmology, i.e. we do not *impose* symmetries by hand)

Are there more local ways of characterizing quantum geometry?

Yes, its “dimension”, which in quantum gravity can behave in unexpected ways.

There are several notions of dimension, which in the Planckian regime need not coincide.

“Dimension” is no longer fixed a priori, but “emerges” from a particular quantum dynamics. It is not pre-determined by the dimensionality of the triangular building blocks used.

As we have already noted, to dynamically generate a 4d extended geometry is highly nontrivial.

$$S^{\mathrm{EH}} = \frac{1}{G_{\mathrm{N}}} \int d^4x \sqrt{-\det g}\,(R[g, \partial g, \partial^2 g] - 2\Lambda)$$

Getting a handle on Planckian physics (via "dimensions")

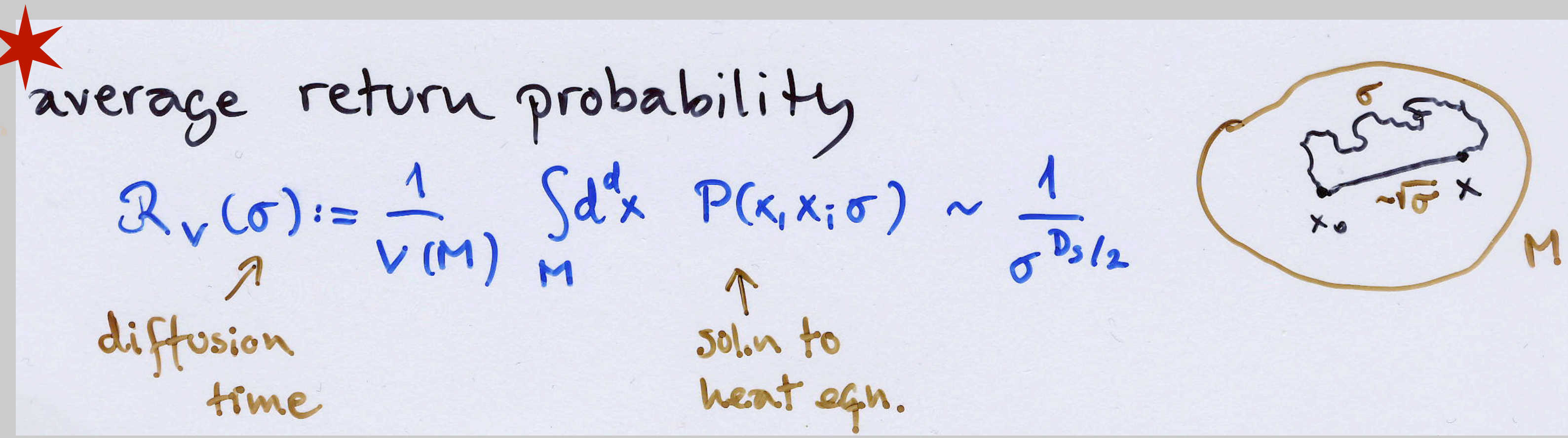
(or, another nonperturbative surprise!)

A diffusion process is sensitive to the dimension of the medium where the "spreading" takes place. We have implemented such a process on the quantum superposition of spacetimes. By measuring a suitable "observable"★, we have extracted the *spectral dimension* D_s of the quantum spacetime.

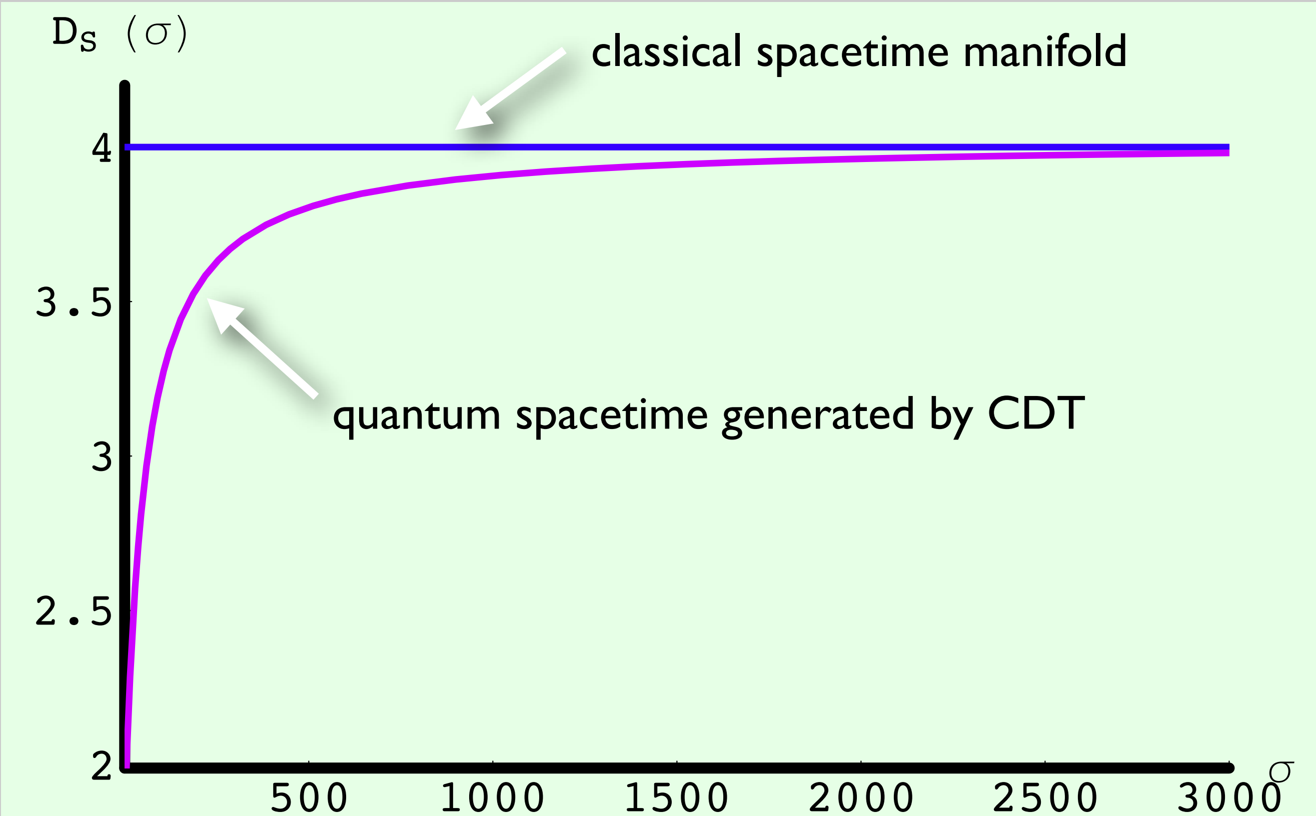
Quite remarkably, we find that it depends on the length scale probed: D_s changes smoothly from 4 on large scales to ~2 on short scales.

(J. Ambjorn, J. Jurkiewicz, RL, PRL 95 (2004) 171301)

★



$D_S(\sigma)$ as probe of geometry on linear length scales ~ $\sigma^{1/2}$



→ on short scales, our "ground state of geometry" is definitely *not* a classical manifold.

Instead, we find evidence for the presence of a random fractal structure.

(J. Ambjørn, A. Görlich, J. Jurkiewicz, RL, PLB 690 (2010) 420)

Intriguingly, the same short-scale "*dynamical dimensional reduction*" has since been found in a couple of disparate, (also quantum field-theoretic) approaches:

- nonperturbative renormalization group flow analysis (M. Reuter, O. Lauscher, JHEP 0510:050, 2005, M. Reuter, F. Saueressig, JHEP 1112 (2011) 012)
- nonrelativistic "Lifshitz quantum gravity" (P. Hořava, PRL 102 (2009) 161301)

Spectral dimension as a tool for understanding quantum gravity

- The spectral dimension $D_S(\sigma)$ is a useful and "covariantly defined" quantity(meaningful in the sum over geometries and after averaging over the starting point of the diffusion process), *and* can be computed (rare in quantum gravity!).

- It can play an important role in discriminating between different candidate theories of quantum gravity, akin to the computation of "black hole entropy" S=A/4, but arguably one that probes the nonperturbative structure, not just semiclassical properties.

- Various computations of $D_S(\sigma)$ on short scales for nonclassical geometries: noncommutative geometry/κ-Minkowski space (D. Benedetti, PRL 102 (2009) 111303), three-dimensional CDT (D. Benedetti, J. Henson, PRD 80 (2009) 124036), from area operator in loop quantum gravity (L. Modesto, CQG 26 (2009) 242002), possible relation with strong-coupling limit of WdW equation (S. Carlip, arXiv: 1009.1136, 1207.4503), modelling from dispersion relations on flat spaces (T. Sotiriou, M. Visser, S. Weinfurtner, PRD 84 (2011) 104018), modelling by multifractal spacetimes (G. Calcagni, PRD 84 (2011) 061501).

Exciting projects currently under investigation ...

- finding renormalization group flow lines in CDT phase diagram for comparison with continuum RG approaches
- determining the effective action in 2+1 dimensions with toroidal slices, going beyond the global conformal mode, and for comparison with anisotropic gravities of Horava-Lifshitz type
- generalized CDT, employing additional building blocks (preserving causality, but relaxing the global time slicing)
- employing Wilson loops as possible observables measuring small- and large-scale curvature

Causal Dynamical Triangulations - Summary & Outlook

CDT is a path integral formulation of gravity, which incorporates the dynamical and causal nature of geometry. It depends on a minimal number of assumptions and ingredients and has few free parameters. Its associated toolbox provides us with an "experimental lab" - a nonperturbative calculational handle on (near-) Planckian physics (c.f. lattice QCD). Time and causal structure must be put in.

- We are able to make quantitative statements about *quantum* geometry (the ground state of quantum gravity).
- We obtained a derivation from first quantum principles of the shape of the universe, illustrating the emergence of classicality from quantum dynamics and the crucial role of "entropy" (number of quantum states).
- We have learned about the totally counterintuitive, dynamical behaviour of "dimension" at the Planck scale, due to the presence of large quantum fluctuation of the spacetime geometry.

$$S^{\rm EH} = \frac{1}{G} \int d^4x \sqrt{-\det g}\,(R[g, \partial g, \partial^2 g] - 2\Lambda)$$

- Hopefully we are seeing glimpses of an essentially unique quantum theory of gravity; are other approaches converging? - Watch this space!

Where to learn more

- CDT light: "The self-organizing quantum universe", by J. Ambjørn, J. Jurkiewicz, RL (Scientific American, July 2008)
- a nontechnical review in Contemp. Phys. 47 (2006) [arxiv: hep-th/0509010]
- recent reviews/lecture notes: arXiv 0906.3947, 1004.0352, 1007.2560, and *our new Physics Report* (in press, arXiv 1203.3591)!
- links to both review and popular science material can be found on my homepage http://www.staff.science.uu.nl/~loll0101

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Bremen, 5 Oct 2012

The End