Consistent Supersymmetric Decoupling in Cosmology.

Work in collaboration with A. Achúcarro.

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Outline of the talk

2 Supergravity
- Gravity as a gauge theory
- Supergravity, the local version of supersymmetry.

3 Constructing supergravity models.
- Scalar manifold.
- Gauge couplings
- Scalar potential
- Overview

4 Supersymmetric Truncations
- The problem
- Methodology
- Summary of the results.

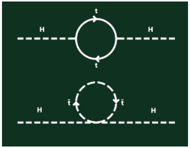
5 Discussion

Supersymmetry and Supergravity

Supersymmetry transforms bosons into fermions and viceversa:

$$Q\,|F>=|B>, \qquad Q\,|B>=|F>,$$

- Every boson must have a fermionic partner.
- Supersymmetry is not seen in nature: the standard model particles can not be fitted in "supermultiplets".
- Supersymmetry might explain *the hierarchy problem* provided supersymmetry is unbroken around the TeV scale.



$$\begin{aligned} M_H &\sim 126\text{ GeV}, \\ \Delta M_H^2 &\sim \Lambda_{GUT}^2 \end{aligned}$$

$\mathcal{N}=1$ Supergravity

Motivation

Supergravity is the local version of supersymmetry.

$$\{Q_\alpha, Q_\beta\} = \tfrac{1}{2}(\gamma_\mu \mathcal{C}^{-1})_{\alpha\beta} P^\mu .$$

Supergravity includes General Relativity

- In general SUSY theories have a better UV behaviour than non-SUSY theories
- Originally supergravity was proposed as a solution to cure the divergences of quantum gravity.
- Today it is mainly regarded as the low energy effective description of a more fundamental theory: *String Theory*.

Part I

Pure Supergravity Action

Field theory in curved space-times

SUSY Decoupling

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Supergravity

Field theory and GR

Local SUSY

The geometry of space-time is dynamical

- The space-time geometry is charaterized by the metric $g_{\mu\nu}$.

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

- The dynamics of $g_{\mu\nu}$ are determined by the Einstein's equations

$$e^{-1}\mathcal{L}_{EH} = -\tfrac{1}{2} R, \qquad R_{\mu\nu} = -8\pi G (T_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} T)$$

Coupling other fields to gravity

$$e^{-1}\mathcal{L} = -\tfrac{1}{2} R - g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\bar{\phi} - \tfrac{1}{2} F_{\mu\nu}F^{\mu\nu} - V(\phi,\bar{\phi}).$$

Gauging a global symmetry

The $U(1)$ symmetry of a complex scalar field can be made local

- The $U(1)$ symmetry acts as a phase shift on the field ϕ:

$$\mathcal{L} = -\partial_\mu \phi \, \partial^\mu \bar{\phi}, \qquad \phi \longrightarrow \phi + \mathrm{i}\lambda(x)\phi$$

- To preserve invariance under local transformations we introduce the covariant derivatives

$$D_\mu \phi \equiv \partial_\mu \phi - \mathrm{i} A_\mu \phi, \qquad A_\mu \longrightarrow A_\mu + \partial_\mu \lambda$$

- the resulting lagrangian is

$$\mathcal{L} = -g^{\mu\nu} D_\mu \phi \, D_\nu \bar{\phi} - \tfrac{1}{2} F_{\mu\nu} F^{\mu\nu}$$

- The Maxwell term is added to capture the dynamics of the gauge boson A_μ.

Gravity as a gauge theory

Gravity can be treated as a gauge theory

- Diffeomorphisms play the röle of gauge transformations. At linear order around Minkowski:

$$\mathcal{L} = -\eta^{mn}\partial_m\phi\partial_n\bar{\phi}, \qquad \phi \longrightarrow \phi + \lambda^\mu(x)\partial_\mu\phi$$

- The corresponding covariant derivative can be defined as

$$D_a\phi \equiv \partial_m\phi - A^\mu_m(\partial_\mu\phi), \qquad A^\mu_m \longrightarrow A^\mu_m + \partial_m\lambda^\mu$$

- We introduce a gauge field A^μ_a for each gauge parameter λ^μ.
- Ordinary derivatives are substituted by covariant derivatives

$$e^{-1}\mathcal{L} = -\eta^{mn}D_m\phi D_n\bar{\phi}$$

Gravity as a gauge theory

Gravity can be treated as a gauge theory

- We can combine the derivative and the gauge terms into one:

$$D_m \equiv \partial_m - A_m^\mu(\partial_\mu) = (\delta_m^\mu - A_m^\mu)\partial_\mu \equiv e_m^\mu \partial_\mu,$$

- we recover the standard lagrangian for ϕ in curved space-time

$$\eta^{mn} D_m \phi D_n \bar{\phi} = g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \bar{\phi}, \qquad g^{\mu\nu} = \eta^{mn} e_m^\mu e_n^\nu$$

- The Einstein-Hilbert term is introduced instead of the Maxwell term to capture the dynamics of e_m^μ:

$$e^{-1}\mathcal{L} = -\frac{1}{2}R - g^{\mu\nu}\partial_\mu \phi \, \partial_\nu \bar{\phi}$$

- This is just the first step: reduce d.o.f. of e_m^μ and include spinors.

Local supersymmetry

Local supersymmetry

- In supersymmetry the gauge parameter ϵ has spin $\frac{1}{2}$

$$\phi \quad \longrightarrow \quad \phi + \delta_\epsilon \phi, \qquad \delta_\epsilon \phi = \epsilon \chi$$

- The corresponding gauge field must transform as

$$\psi_\mu \quad \longrightarrow \quad \psi_\mu + \partial_\mu \epsilon(x) + \ldots$$

- The theory must contain a spin$-\frac{3}{2}$ field ψ_μ, **the gravitino**.

Local supersymmetry implies invariance under local translations

- *Supergravity includes General Relativity*

$$(\delta_\epsilon \delta_\eta - \delta_\eta \delta_\epsilon)\phi \sim \partial_\mu \phi$$

- The theory must contain a spin-2 field e_m^μ, **the vielbein**.

Local supersymmetry
Pure supergravity action

Gravity multiplet $(\frac{3}{2},2)$	vielbein	e_μ^m	$\mu, m = 0, \cdots, 3$
	gravitino	ψ_μ	

- The simplest supergravity theory contains $\mathcal{L}_{EH}$ and the kinetic term for the gravitino (Rarita-Schwinger term):

$$e^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\rho\sigma}D_\rho\psi_\sigma + \mathcal{O}(\psi^4)$$

- Extra terms have to be added so that the lagrangian is invariant under the supersymmetry transformations

$$\begin{aligned} \delta_\epsilon e_\mu^a &= -\frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu, \\ \delta_\epsilon\psi_\mu &= D_\mu\epsilon. \end{aligned}$$

- In general invariance under supersymmetry transformations constraints the type of interactions between fields.

Extended supergravity

- We have discussed minimal supergravity.
- A theory invariant under $\mathcal{N} > 1$ supersymmetries is called *extended supergravity*.
- The corresponding supergravity theory would have $\mathcal{N}$ gravitini.
- Extended supergravity theories are very constrained.
    - Many string compactifications are described with extended supergravity theories.
    - $\mathcal{N} = 1$ supergravity is closer to phenomenology.

Part II

Constructing $\mathcal{N} = 1$ Supergravity Models.

Bosonic sector of the action

Chiral multiplets $(0, \frac{1}{2})$	scalars	ϕ^I	$I = 1, \ldots, n_C$
	chiralini	χ^I	
Vector multiplet $(\frac{1}{2}, 1)$	gauge fields	A_μ^a	$a = 1, \ldots, n_V$
	gaugini	λ^a	

- For simplicity we will focus on the bosonic part of the action:

$$\chi^I = 0, \qquad \lambda^a = 0, \qquad \psi_\mu = 0.$$

Bosonic sector of the action

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$$\chi^I = 0, \qquad \lambda^a = 0, \qquad \psi_\mu = 0.$$

The supergravity lagrangian must include the Einstein-Hilbert term

$$e^{-1}\mathcal{L}_B = -\frac{1}{2}R + \ldots$$

Bosonic sector of the action

Coupling scalar fields

- We can couple several complex scalar fields ϕ^I in the usual way

$$e^{-1}\mathcal{L} = -\frac{1}{2}R - \partial_\mu\phi^1\partial^\mu\bar{\phi}^1 - \partial_\mu\phi^2\partial^\mu\bar{\phi}^2 \dots$$

Bosonic sector of the action

Coupling scalar fields

- We can couple several complex scalar fields ϕ^I in the usual way

$$e^{-1}\mathcal{L} = -\frac{1}{2}R - \delta_{I\bar{J}}\,\partial_\mu\phi^I\partial^\mu\phi^{\bar{J}}\ldots$$

Bosonic sector of the action
Coupling scalar fields

- We can couple several complex scalar fields ϕ^I in the usual way

$$e^{-1}\mathcal{L} = -\frac{1}{2}R - G_{I\bar{J}}(\phi,\bar{\phi})\,\partial_\mu\phi^I\partial^\mu\phi^{\bar{J}}\ldots$$

- Supersymmetric theories admit more general kinetic terms.
- In general, the kinetic terms are characterized by a non-linear sigma model.

Bosonic sector of the action
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- Supersymmetric theories admit more general kinetic terms.
- In general, the kinetic terms are characterized by a non-linear sigma model.

Non-linear sigma models have a simple geometrical interpretation:

- The scalar fields ϕ^I can be seen as coordinates of a manifold $\mathcal{M}$.
- The function $G_{I\bar{J}}(\phi,\bar{\phi})$ is the metric on that manifold.

Bosonic sector of the action
Coupling scalar fields

SUSY Decoupling

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SUGRA models

Scalar manifold.

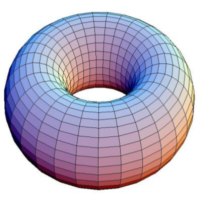
Gauge couplings

Scalar potential

Overview

STEP 1: Choose the shape of the scalar manifold $\mathcal{M}$

$$e^{-1}\mathcal{L} = -\frac{1}{2}R - G_{I\bar{J}}(\phi,\bar{\phi})\,\partial_\mu\phi^I\partial^\mu\phi^{\bar{J}}\ldots$$



- Requiring that

$$G_{I\bar{J}}(\phi,\bar{\phi}) = \partial_I\partial_{\bar{J}}K(\phi,\bar{\phi}),$$
$$\partial_I \equiv \frac{\partial}{\partial\phi^I}.$$

- Supersymmetry requires $\mathcal{M}$ to be **Kahler-Hodge**.
- The metric can be written in terms of the *Kähler potential* $K(\phi,\bar{\phi})$.

Bosonic sector of the action

Coupling scalar fields

- Example 1: **The complex plane**, $K(\phi, \bar{\phi}) = \phi\bar{\phi}$:
  - It represents the canonical kinetic terms:

$$e^{-1}\mathcal{L}_B = -\tfrac{1}{2}R - \partial_\mu\phi\partial^\mu\bar{\phi}$$

$$G_{\phi\bar{\phi}} = \partial_\phi\partial_{\bar{\phi}}K = 1, \quad ds^2 = d\phi d\bar{\phi} = dx^2 + dy^2 \quad (\phi{=}x{+}\mathrm{i}y)$$

Bosonic sector of the action

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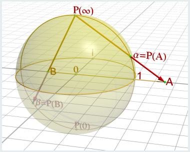
- Example 2: **The sphere**, $K(z,\bar{z}) = -\log(1+\phi\bar{\phi})$:



$$e^{-1}\mathcal{L}_B = -\tfrac{1}{2}R - \frac{\partial_\mu\phi\partial^\mu\bar{\phi}}{(1+\phi\bar{\phi})^2}$$

$$G_{\phi\bar{\phi}} = \partial_\phi\partial_{\bar{\phi}}K = \frac{1}{(1+\phi\bar{\phi})^2}, \qquad ds^2 = \frac{d\phi d\bar{\phi}}{(1+\phi\bar{\phi})^2}$$

Bosonic sector of the action

Gauging global symmetries

STEP 2: Choose which symmetries are promoted to local

- The isometries of the scalar manifold are *global* symmetries of the lagrangian:

$$e^{-1}\mathcal{L} = -\tfrac{1}{2}R - G_{I\bar{J}}(\phi,\bar{\phi})\,\partial_\mu\phi^I\partial^\mu\phi^{\bar{J}}\ldots, \qquad \phi^I \longrightarrow \phi^I + \alpha^a k_a^I(\phi)$$

- $k_a^I(\phi)$ are the *holomorphic killing vectors*.

- To make these symmetries *local* we define the covariant derivatives. In the abelian case:

$$D_\mu\phi^I = \partial_\mu\phi^I - k_a^I A_\mu^a, \qquad A_\mu^a \longrightarrow A_\mu^a + \partial_\mu\alpha^a$$

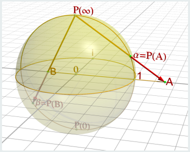
- The choice of killing vectors determines the gauge group $\mathbb{G}$
- If the killing vectors do not commute the gauge group must be non-abelian:

$$[k_a, k_b] = f_{ab}^c k_c.$$

- Example: **The sphere**:

$$e^{-1}\mathcal{L}_B = -\tfrac{1}{2}R - \frac{D_\mu\phi D^\mu\bar{\phi}}{(1+\phi\bar{\phi})^2}$$

The symmetries of the sphere are rotations around the three axes:



$$\begin{aligned} \phi &\longrightarrow \phi + \mathrm{i}\alpha_1\phi \\ \phi &\longrightarrow \phi + \tfrac{1}{2}\alpha_2\,(1+\phi^2) \\ \phi &\longrightarrow \phi + \tfrac{1}{2}\alpha_3\,(1-\phi^2) \end{aligned}$$

- If we make only α_2 local the gauge group is $\mathrm{U}(1)$ and:

$$k_2(\phi) = \tfrac{1}{2}(1+\phi^2), \qquad D_\mu\phi = \partial_\mu\phi - \tfrac{1}{2}(1+\phi^2)A_\mu$$

- If we make local α_1, α_2, and α_3 then $\mathbb{G} = \mathrm{SU}(2)$.

Bosonic sector of the action

Kinetic terms of the gauge sector

STEP 3: Add the kinetic terms for the gauge bosons:

$$e^{-1}\mathcal{L}_B \quad = \quad -\tfrac{1}{2}R - G_{I\bar{J}}\, D_\mu \phi^I D^\mu \phi^{\bar{J}} - \frac{1}{4} F^a{}_{\mu\nu} F^{a\mu\nu}$$

- The field strenghts of the gauge bosons are given by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - f^a_{bc} A^b_\mu A^b_\mu .$$

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- The **kinetic terms of the gauge bosons** are characterized by the holomorphic *gauge kinetic functions*: $f_{ab}(\phi)$.
  - In simple models $\mathrm{Re}\, f_{ab}$ reduces to the Cartan metric of the gauge group.

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Bosonic sector of the action

The scalar potential

STEP 4: Choose a scalar potential:

$$e^{-1}\mathcal{L}_B = -\tfrac{1}{2}R - G_{I\bar{J}}\, D_\mu\phi^I D^\mu\phi^{\bar{J}} - \frac{1}{4}(\mathrm{Re}\, f_{ab})F^a{}_{\mu\nu}F^{b\mu\nu} + \frac{1}{4\sqrt{-g}}(\mathrm{Im}\, f_{ab})F^a{}_{\mu\nu}\epsilon^{\mu\nu\rho\sigma}F^b{}_{\rho\sigma} - V(\phi,\bar{\phi}).$$

- In the absence of gauge couplings the **scalar potential** is determined by the Kähler potential $K(\phi,\bar{\phi})$ and a holomorphic function, the *superpotential* $W(\phi)$:

$$V = \mathrm{e}^K\left(G^{I\bar{J}}\mathcal{D}_I W\mathcal{D}_{\bar{J}}\bar{W} - 3|W|^2\right), \quad \mathcal{D}_I W \equiv \partial_I W - \partial_I K\, W.$$

- Gauge couplings require an extra contribution determined by the killing vectors and the gauge kinetic functions.

Bosonic sector of the action

Final form of the lagrangian

Bosonic sector of the lagrangian reads:

$$e^{-1}\mathcal{L}_B = -\tfrac{1}{2}R - G_{I\bar{J}}\, D_\mu\phi^I D^\mu\phi^{\bar{J}} - \frac{1}{4}(\mathrm{Re}\, f_{ab})F^a{}_{\mu\nu}F^{b\mu\nu} + \frac{1}{4\sqrt{-g}}(\mathrm{Im}\, f_{ab})F^a{}_{\mu\nu}\epsilon^{\mu\nu\rho\sigma}F^b{}_{\rho\sigma} - V(\phi,\bar{\phi}).$$

Summarizing, the action is determined by the following *four* items:

1. The geometry of *the scalar manifold*, encoded in $K(\phi,\bar{\phi})$.
2. the choice of *local symmetries*, $k_a^I(\phi)$ (which fixes $\mathbb{G}$).
3. the *kinetic terms of the gauge bosons*, defined by $f_{ab}(\phi)$,
4. and the scalar potential, determined by *the superpotential* $W(\phi)$.

Part III

Consistent Supersymmetric Truncations

Introduction

Truncations in field theory cosmological models

- Cosmological models based on supersymmetric GUT's and Superstrings typically involve a large number of scalar fields.
- In order to **gain control** and be able to make predictions it is useful to find ways to simplify the models, leaving only a few scalar fields.
- The truncated sector usually consists on those fields stabilized with a large mass.

Introduction

Example: **INFLATION**

- Inflation can be due to the potential energy of a scalar field rolling down a very flat potential:

$$\mathcal{H} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi) \approx V(\phi_0).$$

- Too many scalar fields make difficult to check the slow roll conditions.
- Single field inflationary models fit well the data, while multifield inflationary models are very constrained.
- Which conditions ensure that inflation can be regarded as "single field" in a multifield framework?.

Introduction

It is convenient to leave supersymmetry unbroken during the integration of heavy fields

- **simplicity:** we can calculate the effective theory more efficiently.
- **phenomenology:** it might provide a solution to the hierarchy problem.

Example: **Flux compactifications**

- Cosmological models based in superstrings involve hundreds of scalar fields, such as *the moduli*.
- In flux compatifications a fraction of the moduli is stabilized in a supersymmetric way leaving behind an effective supergravity theory. *Giddings 02*

Which conditions allow the supersymmetric integration of a heavy sector?.

Supersymmetric Truncations in Supergravity

- We would like to understand which type of couplings allow for the supersymmetric decoupling of a heavy sector in $\mathcal{N} = 1$ supergravity.

We study the conditions needed to truncate a heavy sector in a $\mathcal{N} = 1$ supergravity model subject to two requirements:

1. The truncated fields, H^α, should not be sourced by the interactions with the surviving sector, L^i:

$$\frac{\delta S|_{H_0}}{\delta L^i} = 0 \quad \Longrightarrow \quad \frac{\delta S}{\delta L^i}\Big|_{H_0} = 0,$$

2. The reduced theory for the low energy fields must be described by $\mathcal{N} = 1$ supergravity.

- These conditions translate into constraints for the couplings between the truncated (heavy) and the surviving (light) sectors.

Supersymmetric Truncations in Supergravity

The truncation is defined by the following two conditions:

- The scalar fields in the heavy sector are fixed at a extremum of the scalar potential with a expectation value

$$H^{\alpha} = H_0^{\alpha}, \qquad S(H, \bar{H}, L, \bar{L}) \longrightarrow S^{light}(L, \bar{L}).$$

In particular this condition defines a submanifold of the Kähler manifold.

- The expectation value of the truncated fields might break some gauge symmetries, when

$$\delta H^{\alpha} = k_{\tilde{a}}^{\alpha}(H_0, L) \neq 0.$$

The gauge bosons associated to broken gauge symmetries must be truncated:

$$F_{\mu\nu}^{\tilde{a}} = 0.$$

Supersymmetric Truncations in Supergravity

Supersymmetry transformations

- In order to find the conditions which allow the supersymmetric decoupling of the heavy sectors, we study the supersymmetry transformations.

Supersymmetry transformations of the chiralini and the gaugini

$$\begin{aligned} \delta\chi^I{}_L &= \tfrac{1}{2}\gamma^\mu\nabla_\mu\phi^I\epsilon_R - \tfrac{1}{2}\,\mathrm{e}^{\frac{1}{2}K}K^{I\bar{J}}\mathcal{D}_{\bar{J}}\bar{W}\,\epsilon_L \\ \delta\lambda^a &= \tfrac{1}{4}\gamma^{\mu\nu}F^a{}_{\mu\nu}\epsilon + \tfrac{1}{2}i(\mathrm{Re}\,f)^{-1|ab}\mathcal{P}_b\,\gamma_5\epsilon. \end{aligned}$$

- The supersymmetry tranformations depend on the couplings between fields: $K(\phi,\bar{\phi})$, $\mathbb{G}$, $f_{ab}(\phi)$, $k^I_a(\phi)$ and $W(\phi)$.

The supersymmetry transformations split into two sets

Supersymmetry transformations of the HEAVY fields: χ^α, $\lambda^{\tilde{a}}$

$$\begin{aligned} \delta\chi^\alpha{}_L &= \tfrac{1}{2}\gamma^\mu \nabla_\mu H^\alpha \epsilon_R - \tfrac{1}{2}\,\mathrm{e}^{\frac{1}{2}K} K^{\alpha\bar{\beta}} \mathcal{D}_{\bar{\beta}} \bar{W}\,\epsilon_L - \tfrac{1}{2}\,\mathrm{e}^{\frac{1}{2}K} K^{\alpha\bar{i}} \mathcal{D}_{\bar{i}} \bar{W}\,\epsilon_L \\ \delta\lambda^{\tilde{a}} &= \tfrac{1}{4}\gamma^{\mu\nu} F^{\tilde{a}}{}_{\mu\nu}\epsilon + \tfrac{1}{2} i(\mathrm{Re}\, f)^{-1|\tilde{a}\tilde{b}} \mathcal{P}_{\tilde{b}}\,\gamma_5\epsilon + \tfrac{1}{2} i(\mathrm{Re}\, f)^{-1|\tilde{a}b} \mathcal{P}_b\,\gamma_5\epsilon. \end{aligned}$$

Supersymmetry transformations of the LIGHT fields: χ^i, λ^a

$$\begin{aligned} \delta\chi^i{}_L &= \tfrac{1}{2}\gamma^\mu \nabla_\mu L^i \epsilon_R - \tfrac{1}{2}\,\mathrm{e}^{\frac{1}{2}K} K^{i\bar{j}} \mathcal{D}_{\bar{j}} \bar{W}\,\epsilon_L - \tfrac{1}{2}\,\mathrm{e}^{\frac{1}{2}K} K^{i\bar{\beta}} \mathcal{D}_{\bar{\beta}} \bar{W} \epsilon_L \\ \delta\lambda^a &= \tfrac{1}{4}\gamma^{\mu\nu} F^a{}_{\mu\nu}\epsilon + \tfrac{1}{2} i(\mathrm{Re}\, f)^{-1|ab} \mathcal{P}_b\,\gamma_5\epsilon + \tfrac{1}{2} i(\mathrm{Re}\, f)^{-1|a\tilde{b}} \mathcal{P}_{\tilde{b}}\,\gamma_5\epsilon. \end{aligned}$$

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\delta\lambda^{\tilde a} &= \tfrac{1}{4}\gamma^{\mu\nu}F^{\tilde a}{}_{\mu\nu}\epsilon + \tfrac{1}{2}i(\mathrm{Re}\,f)^{-1|\tilde a\tilde b}\mathcal{P}_{\tilde b}\,\gamma_5\epsilon + \tfrac{1}{2}i(\mathrm{Re}\,f)^{-1|\tilde a b}\mathcal{P}_b\,\gamma_5\epsilon.
\end{aligned}$$

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\delta\lambda^{\tilde a} &= \tfrac{1}{4}\gamma^{\mu\nu}F^{\tilde a}{}_{\mu\nu}\epsilon + \tfrac{1}{2}i(\mathrm{Re}\,f)^{-1|\tilde a\tilde b}\mathcal{P}_{\tilde b}\,\gamma_5\epsilon + \tfrac{1}{2}i(\mathrm{Re}\,f)^{-1|\tilde a b}\mathcal{P}_b\,\gamma_5\epsilon.
\end{aligned}$$

Supersymmetry transformations of the LIGHT fields: χ^i, λ^a

$$\begin{aligned}
\delta\chi^i{}_L &= \tfrac{1}{2}\gamma^\mu\nabla_\mu L^i\epsilon_R - \tfrac{1}{2}\,\mathrm{e}^{\frac{1}{2}K}K^{i\bar j}\mathcal{D}_{\bar j}\bar W\,\epsilon_L - \tfrac{1}{2}\,\mathrm{e}^{\frac{1}{2}K}K^{i\bar\beta}\mathcal{D}_{\bar\beta}\bar W\epsilon_L\\
\delta\lambda^a &= \tfrac{1}{4}\gamma^{\mu\nu}F^a{}_{\mu\nu}\epsilon + \tfrac{1}{2}i(\mathrm{Re}\,f)^{-1|ab}\mathcal{P}_b\,\gamma_5\epsilon + \tfrac{1}{2}i(\mathrm{Re}\,f)^{-1|a\tilde b}\mathcal{P}_{\tilde b}\,\gamma_5\epsilon.
\end{aligned}$$

- The truncated sector must preserve supersymmetry.
- The supersymmetry transformations of the light sector should reduce to the form required by SUGRA if there was no heavy sector.

It is sufficient to solve the following constraints

$$\begin{aligned}
\tfrac{1}{2}\gamma^\mu \nabla_\mu \chi^\alpha \epsilon_R - \tfrac{1}{2}\,\mathrm{e}^{\frac{1}{2}K} K^{\alpha\bar\beta}\mathcal{D}_{\bar\beta}\bar W\,\epsilon_L - \tfrac{1}{2}\,\mathrm{e}^{\frac{1}{2}K} K^{\alpha\bar i}\mathcal{D}_{\bar i}\bar W\,\epsilon_L &= 0,\\
\tfrac{1}{4}\gamma^{\mu\nu} F^{\tilde a}{}_{\mu\nu}\epsilon + \tfrac{1}{2} i(\mathrm{Re}\, f)^{-1|\tilde a\tilde b}\mathcal{P}_{\tilde b}\,\gamma_5\epsilon + \tfrac{1}{2} i(\mathrm{Re}\, f)^{-1|\tilde a b}\mathcal{P}_b\,\gamma_5\epsilon &= 0,
\end{aligned}$$

for any arbitrary configuration of the fields L^i of the reduced theory.

- If the conditions we find are not preserved by supersymmetry we impose new constraints until we reach self-consistency.
  Andranopoli 01

Summary of the results

FIELD CONTENT

- Each scalar field H_0^α must be truncated with its whole supermultiplet:

$$H^\alpha = H_0^\alpha, \qquad \chi^\alpha = 0.$$

- Gauge fields associated to broken symmetries $k_{\tilde{a}}^\alpha(H_0, L) \neq 0$ must be truncated with their supermultiplets:

$$F_{\mu\nu}^{\tilde{a}} = 0, \qquad \lambda^{\tilde{a}} = 0.$$

Summary of the results

The **KINETIC TERMS** of the truncated and surviving fields must be decoupled:

- The scalar manifold of the reduced theory is a **totally geodesic Kähler submanifold** of the parent manifold,
- thus, the sigma model metric is block diagonal at $H^\alpha = H_0^\alpha$:

$$G|_{(H_0,L)} = \begin{pmatrix} G^h & 0 \\ 0 & G^l \end{pmatrix} \quad \text{for all } L^i.$$

- The real part of the gauge kinetic functions should be block diagonal in the truncated and suriving gauge fields:

$$Re\, f|_{(H_0,L)} = \begin{pmatrix} Re\, f^h & 0 \\ 0 & Re\, f^l \end{pmatrix}, \quad \text{and} \quad Re\,(\partial_\alpha f^l)|_{(H_0,L)} = 0.$$

Summary of the results

The GAUGE INTERACTIONS should respect the truncation

- Non-abelian interactions should not source the truncated gauge bosons. If the gauge group $\mathbb{G}$ is semi-simple, and $\mathbb{G}_h$ is the broken subgroup

$$\mathbb{G} = \mathbb{G}_h \times \mathbb{G}_l \quad \text{otherwise} \quad \mathbb{G}_l \lhd \mathbb{G}.$$

- Truncated gauge bosons should not be sourced by the surviving fields in the chiral multiplets:

$$k^i_{\tilde{a}}(H_0, L) = 0, \qquad k^\alpha_{\tilde{a},i}(H_0, L) = 0 \quad \text{for all } L^i.$$

- Truncated fields in the chiral multiplets should not be sourced by the surviving gauge bosons:

$$k^\alpha_a(H_0, L) = 0, \qquad k^i_{a,\alpha}(H_0, L) = 0 \quad \text{for all } L^i.$$

Summary of the results

SCALAR POTENTIAL

- In addition, the superpotential must satisfy the constraint $\mathcal{D}_\alpha W|_{H_0} = 0$, which is solved locally by

$$W = W_0(\phi)\mathrm{e}^{-\gamma_\alpha h^\alpha},$$

  where $W_0(\phi)$ is an arbitrary holomorphic function, the equations $h^\alpha(\phi) = 0$ define the reduced scalar manifold, and γ_α is determined by W_0 and h^α.

- This ensures that the Hessian of the scalar potential V is block diagonal in the truncated and surviving sectors at $H^\alpha = H_0^\alpha$

$$V|_{(H_0,L)} = \begin{pmatrix} V^h & 0 \\ 0 & V^l \end{pmatrix} \quad \text{for all } L^i.$$

Discussion

- We opresented the conditions required for truncating a heavy sector in $\mathcal{N} = 1$ SUGRA subject to two requirements:
  1. the heavy fields are not sourced by the low energy fields.
  2. supersymmetry is exactly preserved.
- These conditions are expressed as constraints on

$$K(\phi, \bar{\phi}), \quad k_a^I(\phi), \quad \mathbb{G}, \quad f_{ab}(\phi) \quad \text{and} \quad W(\phi).$$

- In particular, this result shows how to couple a working inflationary model to a heavy sector without spoiling the slow-roll conditions.
- Consistency also requires the heavy field configuration to be perturbatively stable. *arXiv: 0712.3460, 0809.1441*