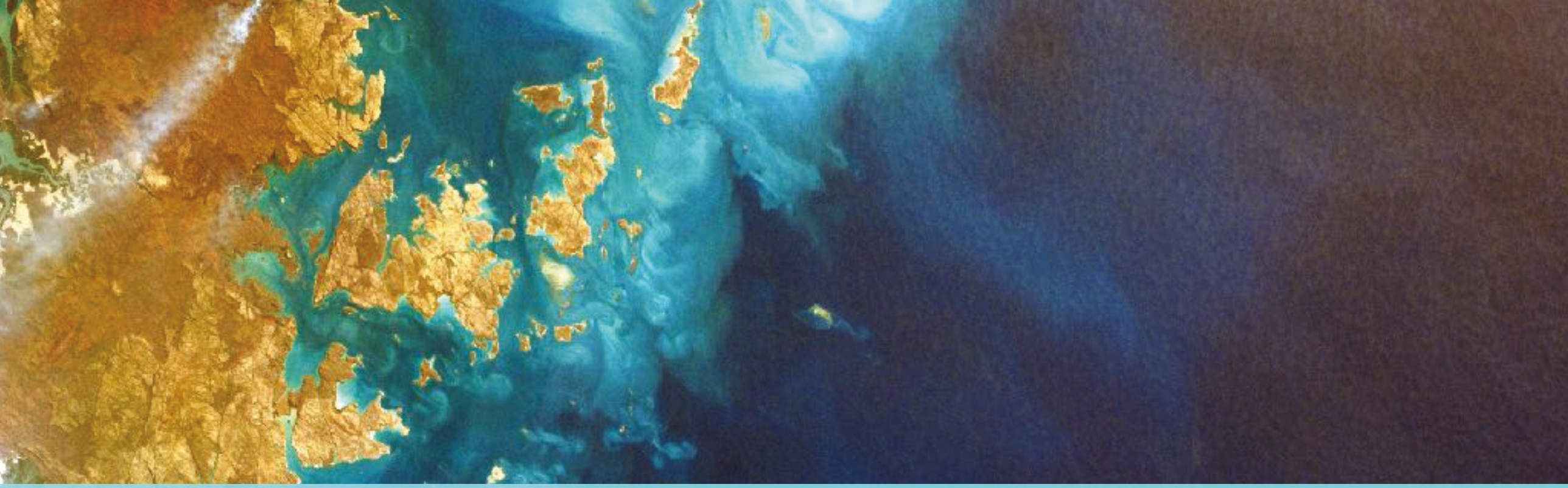
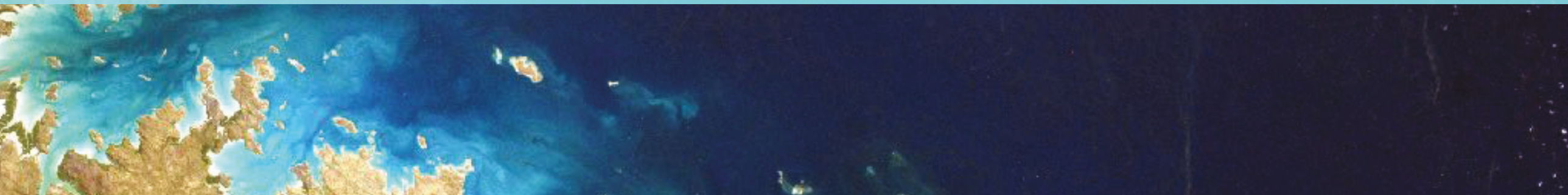


TESTS OF GRAVITY WITH GRAVITATIONAL WAVES & COSMOLOGY

TESSA BAKER, UNIVERSITY OF OXFORD



OUTLINE

- The EFT of cosmological perturbations.
- The impact of GW170817.
- Constraints from cosmology & summary.


Image: Tom Barrett

THE EFT OF COSMOLOGICAL PERTURBATIONS

LAGRANGIAN EXPANSION

1. Let Θ be a vector of fields mediating gravity.
2. Taylor expand the gravitational Lagrangian in perturbations δΘ:

$$L \simeq \bar{L} + L_{\Theta_a}\,\delta\Theta_a + \frac{1}{2}\,L_{\Theta_a\Theta_b}\delta\Theta_a\delta\Theta_b + \ldots$$

$$\frac{\partial L}{\partial \Theta_a}$$

Gives us linearised grav. field equations.

Scalar perturbations of metric:

$$ds^2 = -\left(1+2\Phi\right)dt^2 + 2\partial_i B\,dx^i\,dt$$
$$+\,a^2(t)\left[\left(1-2\Psi\right)\delta_{ij} + 2\partial_i\partial_j E\right]dx^i\,dx^j$$

LAGRANGIAN EXPANSION

3. Build Lagrangian containing all combinations of fields in Θ.

   Simplest case: $\Phi,\ \Psi,\ B,\ E,\ \phi.$

$$\delta_2 L = L_{\Phi\Phi}\,\Phi^2 + L_{\Phi\Psi}\,\Phi\,\Psi + \ldots$$

$$+ L_{\dot\Psi\dot\Psi}\,\dot\Psi^2 + L_{\dot\Psi\,\partial^2 B}\,\dot\Psi\,\partial_i\partial^i B + \ldots$$

$$+ L_{\dot\phi\dot\phi}\,\delta\dot\phi^2 + L_{\phi\partial^2\dot E}\,\delta\phi\,\partial_i\partial^i\dot E + \ldots$$

$+$ usual fluid matter sector

~ 70 terms in total.

SYMMETRIES AND CONSEQUENCES

4. The action must be coordinate-invariant.

$\Rightarrow$ Enforce linear diff symmetry: $x^\mu \to x^\mu + \epsilon^\mu$

$$\epsilon^\mu = \left(\pi,\, \partial^i \epsilon\right)$$

$$\Rightarrow \quad \delta_2 S \to \delta_2 S + \underbrace{\left[\begin{array}{l}\text{terms linear in} \\ \delta\varphi,\, \boldsymbol{\phi},\, \boldsymbol{\psi} \text{ etc.}\end{array}\right] \times \left(\pi \text{ or } \epsilon\right)}_{\text{must vanish}}$$

Invariance of action under a ***non-dynamical*** symmetry gives a set of constraint relations.

SYMMETRIES AND CONSEQUENCES

4. The action must be coordinate-invariant.

Invariance of action under a ***non-dynamical*** symmetry gives a set of constraint relations.

$$\delta_2 L = L_{\Phi\Phi}\,\Phi^2 + L_{\Phi\Psi}\,\Phi\,\Psi + \ldots$$

$$+ L_{\dot\Psi\dot\Psi}\,\dot\Psi^2 + L_{\dot\Psi\,\partial^2 B}\,\dot\Psi\,\partial_i\partial^i B + \ldots$$

$$+ L_{\dot\phi\dot\phi}\,\delta\dot\phi^2 + L_{\phi\partial^2\dot E}\,\delta\phi\,\partial_i\partial^i\dot E + \ldots$$

$+$ usual fluid matter sector

SYMMETRIES AND CONSEQUENCES

4. The action must be coordinate-invariant.

Invariance of action under a ***non-dynamical*** symmetry gives a set of constraint relations. E.g. :

$$L_{\Phi\Phi} + 3\dot{H}\, L_{\Phi\Psi} = 0$$

$$L_{\dot{\phi}\dot{\phi}} - L_{\dot{\Psi}\dot{\Psi}} + 5\, L_{\phi\partial^2\dot{E}} = 0$$

etc.

⇒ Solve system constraint equations.

⇒ Reduce to a handful of `true' free functions.

THE ALPHA PARAMETERS

$\alpha_T(t)$: speed of gravitational waves, $c_T^2 = 1 + \alpha_T$.

$\alpha_K(t)$: kinetic term of scalar field.

$\alpha_B(t)$: `braiding' – mixing of scalar + metric kinetic terms.

$\alpha_M(t) = \dfrac{1}{H}\dfrac{d\ln M^2(t)}{dt}$: running of effective Planck mass.

$\alpha_H(t)$: disformal symmetries of the metric.

THE ALPHA PARAMETERS

Bellini & Sawicki, 1404.3713

Model Class		α_K	α_B	α_M	α_T
ΛCDM		***0***	***0***	***0***	***0***
quintessence	[1, 2]	$(1-\Omega_m)(1+w_X)$	0	0	0
k-*essence*/perfect fluid	[45, 46]	$\frac{(1-\Omega_m)(1+w_X)}{c_s^2}$	0	0	0
kinetic gravity braiding	[47–49]	$\frac{m^2(n_m+\kappa_\phi)}{H^2 M_{Pl}^2}$	$\frac{m\kappa}{HM_{Pl}^2}$	0	0
galileon cosmology	[57]	$-\frac{3}{2}\alpha_M^3 H^2 r_c^2 e^{2\phi/M}$	$\frac{\alpha_K}{6}-\alpha_M$	$\frac{-2\dot\phi}{HM}$	0
BDK	[26]	$\frac{\dot\phi^2 K_{,\dot\phi\dot\phi} e^{-\kappa}}{H^2 M^2}$	$-\alpha_M$	$\frac{\dot\kappa}{H}$	0
metric $f(R)$	[3, 72]	0	$-\alpha_M$	$\frac{B\dot H}{H^2}$	0
MSG/Palatini $f(R)$	[73, 74]	$-\frac{3}{2}\alpha_M^2$	$-\alpha_M$	$\frac{2\dot\phi}{H}$	0
f(Gauss-Bonnet)	[52, 75, 76]	0	$\frac{-2H\dot\xi}{M^2+H\dot\xi}$	$\frac{\dot H\dot\xi+H\ddot\xi}{H(M^2+H\dot\xi)}$	$\frac{\ddot\xi-H\dot\xi}{M^2+H\dot\xi}$

Lagos+, 1711.09893

VECTOR-TENSOR RESULTS

We can play the game over again with a vector field.

$$A^\mu \sim \left(\bar{A} - \delta A_0,\, \partial^i \delta A_1\right)$$

Results:

$\alpha_T(t)$, $\alpha_K(t)$, $\alpha_M(t)$ as for the scalar case.

$\alpha_V(t)$ vector mass mixing ~ $\delta A_0\, \delta A_1$

$\alpha_D(t)$ small-scale dynamics ~ $k^4\, \delta A_1^2$

$\alpha_A(t)$ `auxiliary friction' ~ $\delta A_0\, \delta \dot{A}_1$

$\alpha_C(t)$ conformal coupling excess ~ changes effective mass scale of theory

RESULTS BREAKDOWN

Fields	# True free functions	Example full theory
$g_{\mu\nu}$	1	GR
$g_{\mu\nu},\ \phi$	5	Horndeski
$g_{\mu\nu},\ A^{\mu}$	7	Generalised Proca
$g_{\mu\nu},\ A^{\mu},\ \lambda$	4	Einstein-Aether
$g_{\mu\nu},\ q_{\mu\nu}$	5	Massive Bigravity
$g_{\mu\nu},\ \phi,\ A^{\mu},\ q_{\mu\nu}$	12	Uber-case

(Some restrictions imposed, e.g. 1 propagating d.o.f.)


Image: Tijana Mihajlovic

THE IMPACT OF GRAVITATIONAL WAVES

THE ALPHA PARAMETERS

$\alpha_T(t)$: speed of gravitational waves, $c_T^2 = 1 + \alpha_T$.

$\alpha_K(t)$: kinetic term of scalar field.

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$\alpha_M(t) = \frac{1}{H}\frac{d\ln M^2(t)}{dt}$: running of effective Planck mass.

$\alpha_H(t)$: disformal symmetries of the metric.

GW170817 & GRB 170817A

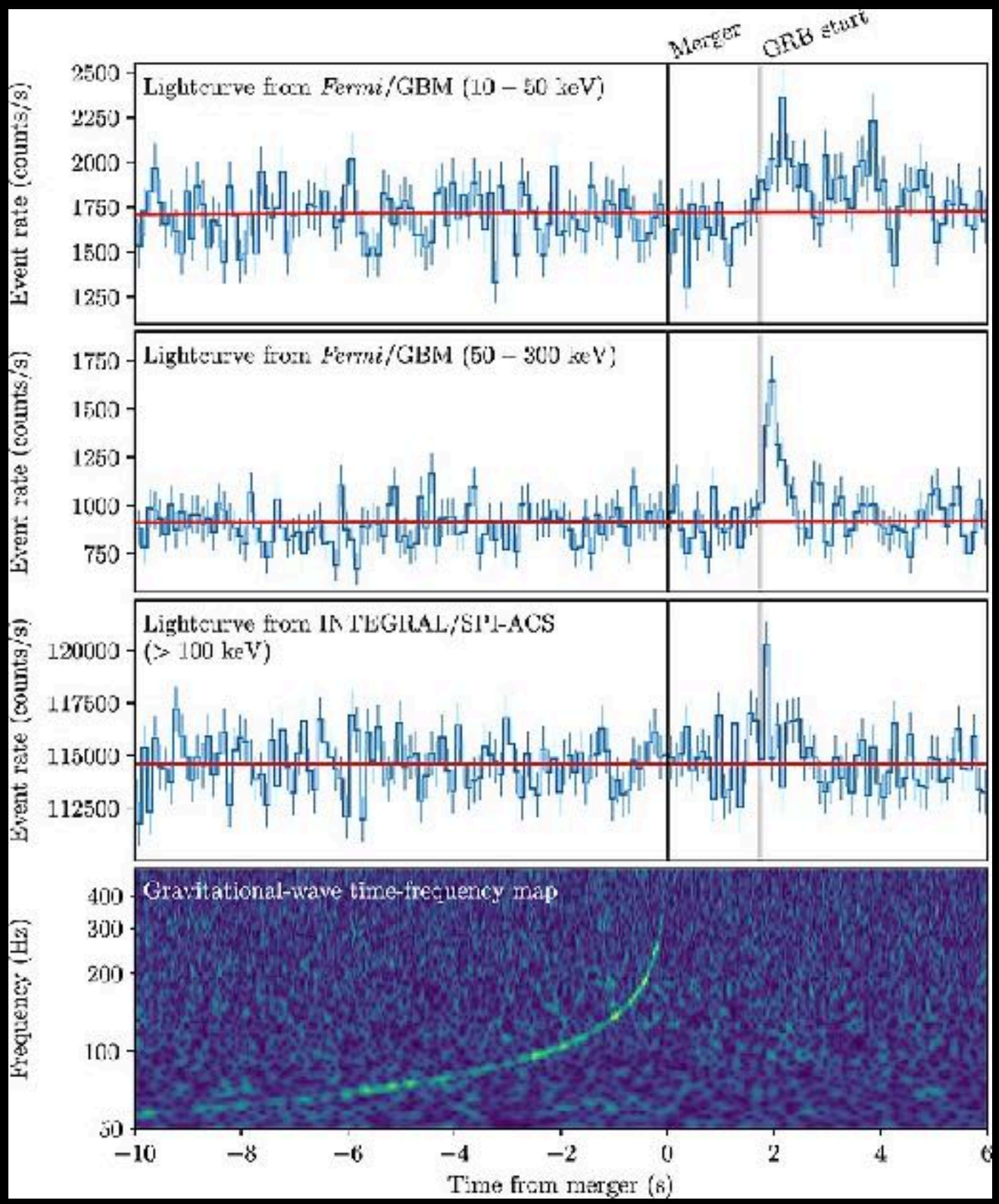


Image: LIGO-VIRGO Collaboration, ApJ 848:2 (2017).

$$c_T^2 = 1 + \alpha_T$$

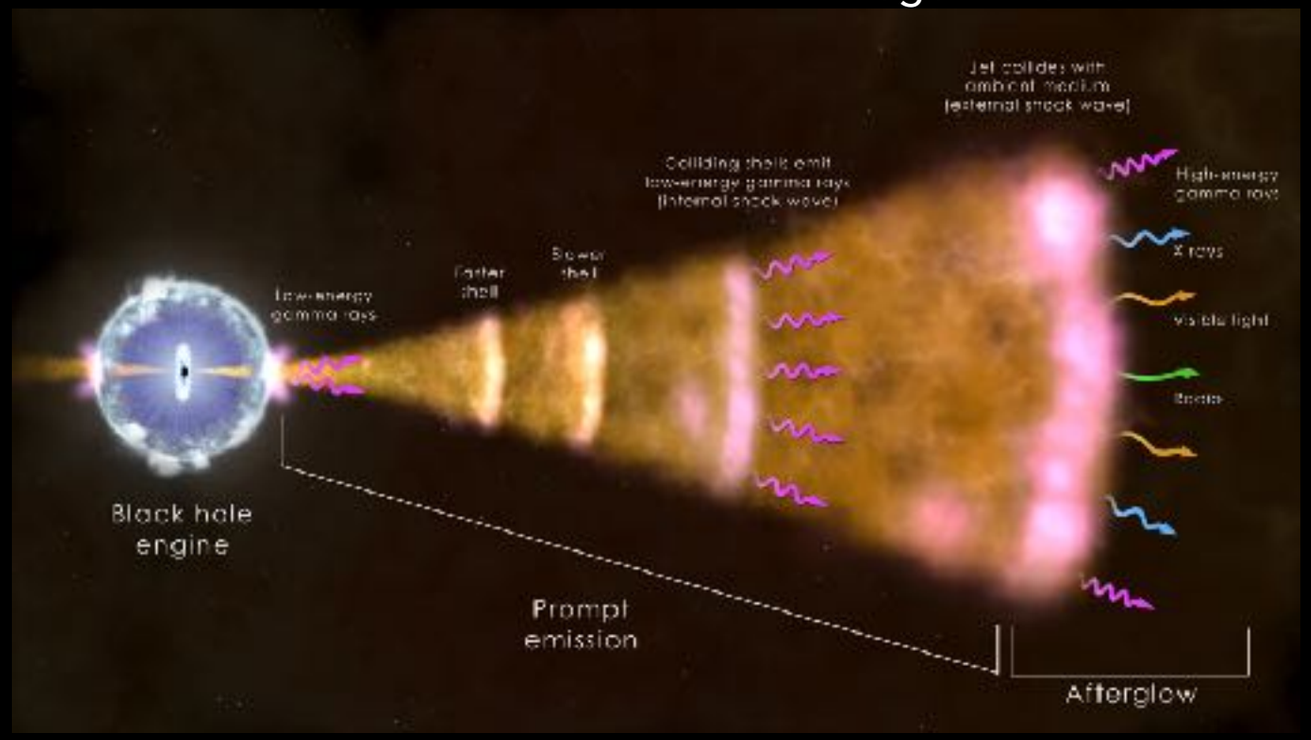
$$\Delta t \simeq 1.7\,\mathrm{s}$$

$$\Rightarrow \quad |\alpha_T| \lesssim 10^{-15}$$

!! c. f. $|\alpha_M|, |\alpha_B| \lesssim \mathcal{O}(1)$

GW170817 & GRB 170817A

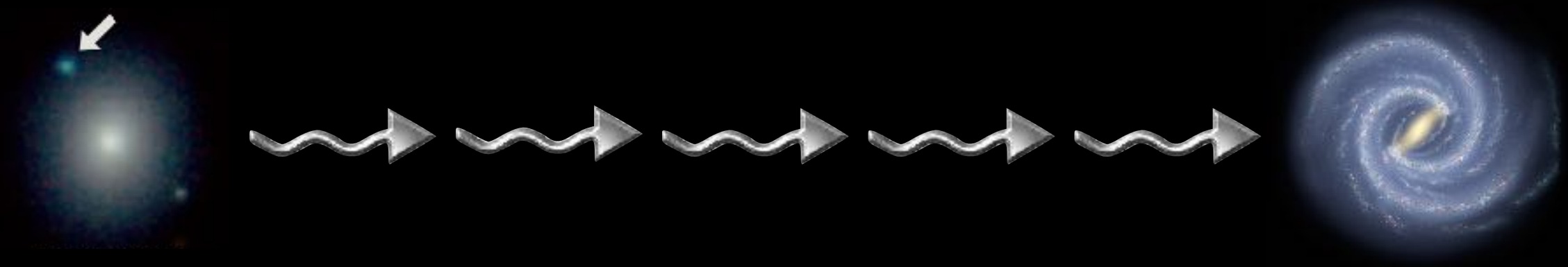
Image: NASA Goddard



Assumptions:

1. No fine-tuned cancellation of *intrinsic emission delay.*

2. Propagation time dominated by cosmological regime.



3. No finely-tuned, protected functional cancellations in theory

1710.06394
1710.05877
1710.05893
1710.05901

GW170817 & GRB 170817A

What does this mean for gravity theories?

Scalar case clearest; full theory is **Horndeski gravity**.

$$S = \int d^4x \sqrt{-g}\ \Sigma_{i=2}^{5}\, \mathcal{L}_i \ + \ S_M$$

$$\mathcal{L}_2 = K \qquad\qquad \mathcal{L}_3 = -G_3 \Box\phi$$

$$\mathcal{L}_4 = G_4 R + G_{4,X} \left\{ (\Box\phi)^2 - \nabla_\mu \nabla_\nu \phi \nabla^\mu \nabla^\nu \phi \right\}$$

$$\mathcal{L}_5 = G_5 G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{1}{6} G_{5,X} \left\{ (\nabla\phi)^3 \right.$$
$$\left. - 3\nabla^\mu \nabla^\nu \phi \nabla_\mu \nabla_\nu \phi \Box\phi + 2\nabla^\nu \nabla_\mu \phi \nabla^\alpha \nabla_\nu \phi \nabla^\mu \nabla_\alpha \phi \right\}$$

where $G_i = G_i(\phi, X)$ and $X = -\nabla_\nu \phi \nabla^\nu \phi / 2$.

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Linearised theory maps to alpha parameters:

$$\Rightarrow\ \alpha_T(t) = \frac{2X}{M_*^2}\left[2G_{4,X} - 2G_{5,\phi} - \left(\ddot{\phi} - \dot{\phi}H\right)G_{5,X}\right]$$

Barring fine-tuned cancellations, $\Rightarrow\ G_{4,X} = G_{5,\phi} = G_{5,X} = 0\,.$

GW170817 & GRB 170817A

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$$\mathcal{L}_4 = G_4 R$$

$$\mathcal{L}_5 = G_5 G_{\mu\nu} \nabla^\mu \nabla^\nu \phi \; = \; 0$$ by Bianchi identity

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GW170817 & GRB 170817A

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Scalar case clearest; full theory is **Horndeski gravity**.

$$\mathcal{L}_4 = G_4 R \qquad \mathcal{L}_2 = K \qquad \mathcal{L}_3 = -G_3 \Box\phi$$

$$\phi = f_R \qquad f(R) - R f_R \qquad 0$$

$\Rightarrow$ f [R] gravity fits the template, so it survives.

GW170817 & GRB 170817A

Heisenberg+
1605.05565

What does this mean for gravity theories?

The vector-tensor equivalent of Horndeski is **Generalised Proca**:

$$\mathcal{L}_1 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad \mathcal{L}_2 = G_2 \qquad \mathcal{L}_3 = G_3\nabla_\mu A^\mu$$

$$\begin{aligned}\mathcal{L}_4 &= G_4 R \\ &\quad + G_{4,X}\left[(\nabla_\mu A^\mu)^2 + c_2\nabla_\rho A_\sigma \nabla^\rho A^\sigma - (1+c_2)\nabla_\rho A_\sigma \nabla^\sigma A^\rho\right]\end{aligned}$$

$$\begin{aligned}\mathcal{L}_5 &= G_5 G_{\mu\nu}\nabla^\mu A^\nu \\ &\quad - \frac{1}{6}G_{5,X}[(\nabla_\mu A^\mu)^3 - 3d_2\nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\rho A^\sigma + \text{similar}\,]\end{aligned}$$

where $G_i = G_i(X)$ and $X = -\frac{1}{2}A_\mu A^\mu$.

GW170817 & GRB 170817A

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$$\mathcal{L}_5 = G_5 G_{\mu\nu}\nabla^\mu A^\nu - \frac{1}{6} G_{5,X}[(\nabla_\mu A^\mu)^3 - 3d_2 \nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\rho A^\sigma + \text{similar}\,]$$

$$\boxed{\alpha_T = \frac{A^2}{\tilde{M}_*^2}\left[2G_{4,X} - (HA - \dot{A})G_{5,X}\right]} \Rightarrow G_{4,X} = G_{5,X} = 0$$

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by Bianchi identity again.

$$\boxed{\alpha_T = \frac{A^2}{\tilde{M}_*^2}\left[2G_{4,X} - (HA - \dot{A})G_{5,X}\right]} \Rightarrow G_{4,X} = G_{5,X} = 0$$

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$$\mathcal{L}_4 = G_4 R$$

de Rham, 1606.08462

GW170817 & GRB 170817A

What does this mean for gravity theories?

For bimetric theories, get a bound on graviton mass:

$$m_g \lesssim 10^{-22}\, eV$$

This is not competitive with existing Solar System bounds:

$$m_g \lesssim 10^{-32}\, eV$$

(from Lunar Laser Ranging & Earth-Moon precession)

Image: Yicai Global


Image: Allen Cai

COSMOLOGICAL OBSERVATIONS

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$\alpha_H(t)$: disformal symmetries of the metric.

CODES

Effective Field Theory with CAMB

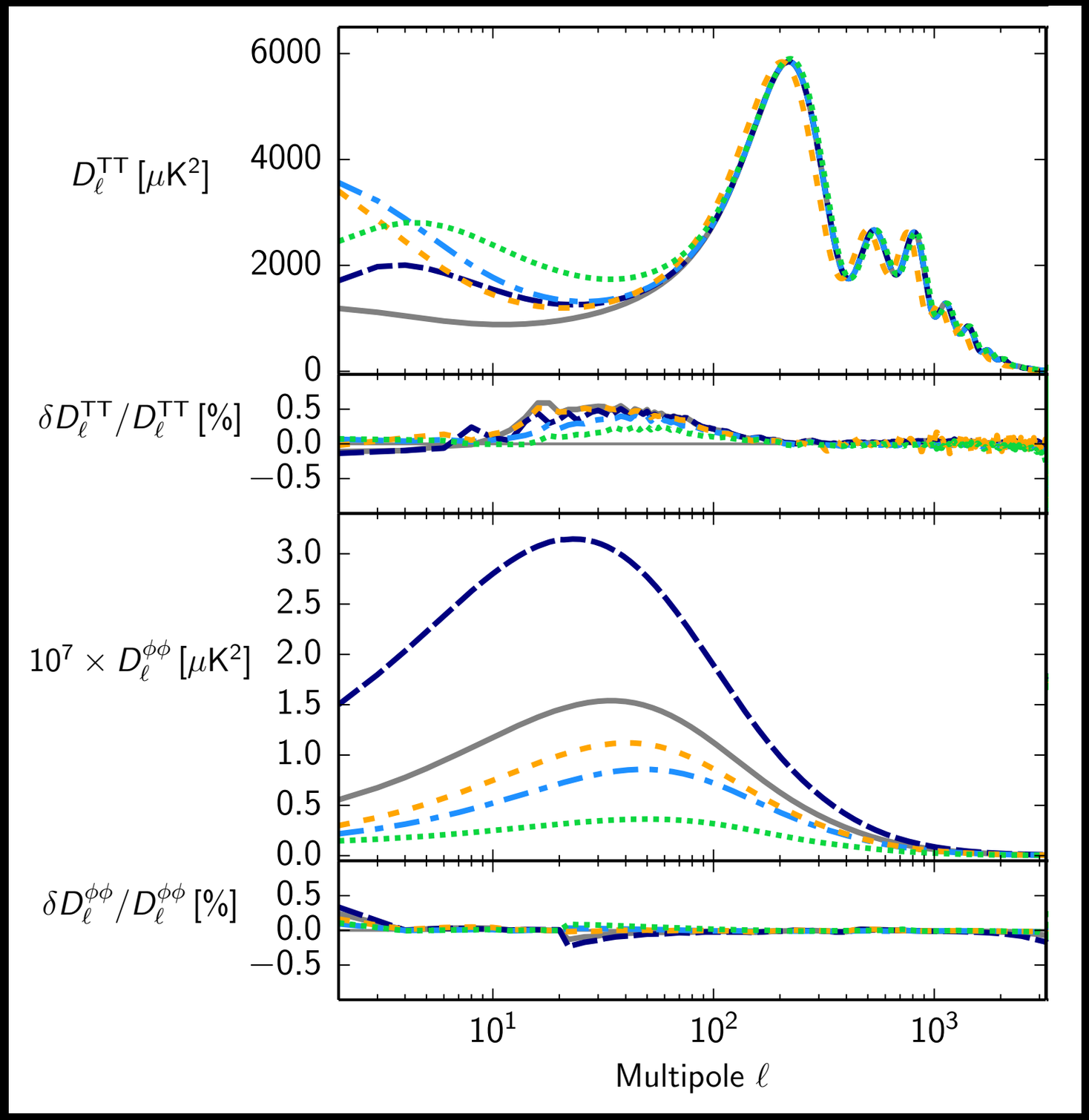
Hu, Raveri, Frusciante & Silvestri (2014).

hi_class

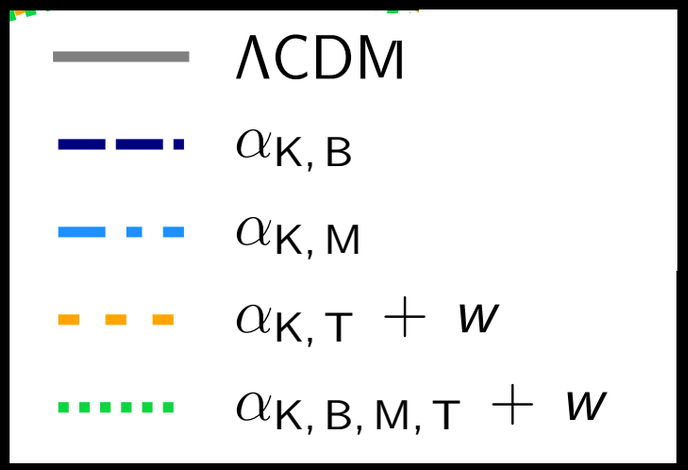
Zumalacarregui, Bellini, Sawicki & Lesgourgues (2016).

Bellini+, 1709.09135

EFFECTS ON OBSERVABLES

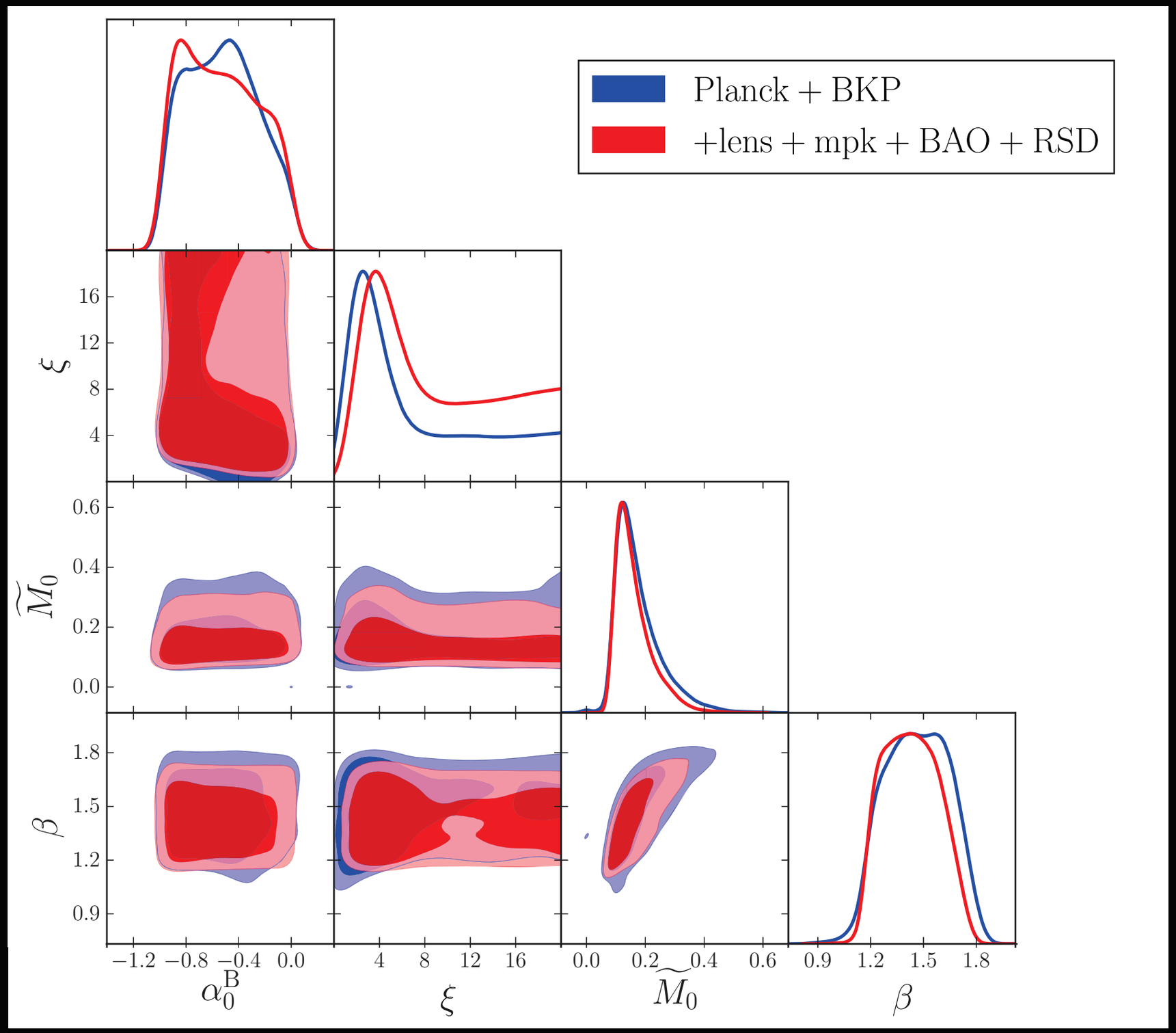


CMB temperature power spectrum.



CMB lensing power spectrum

THE CURRENT STATE OF PLAY



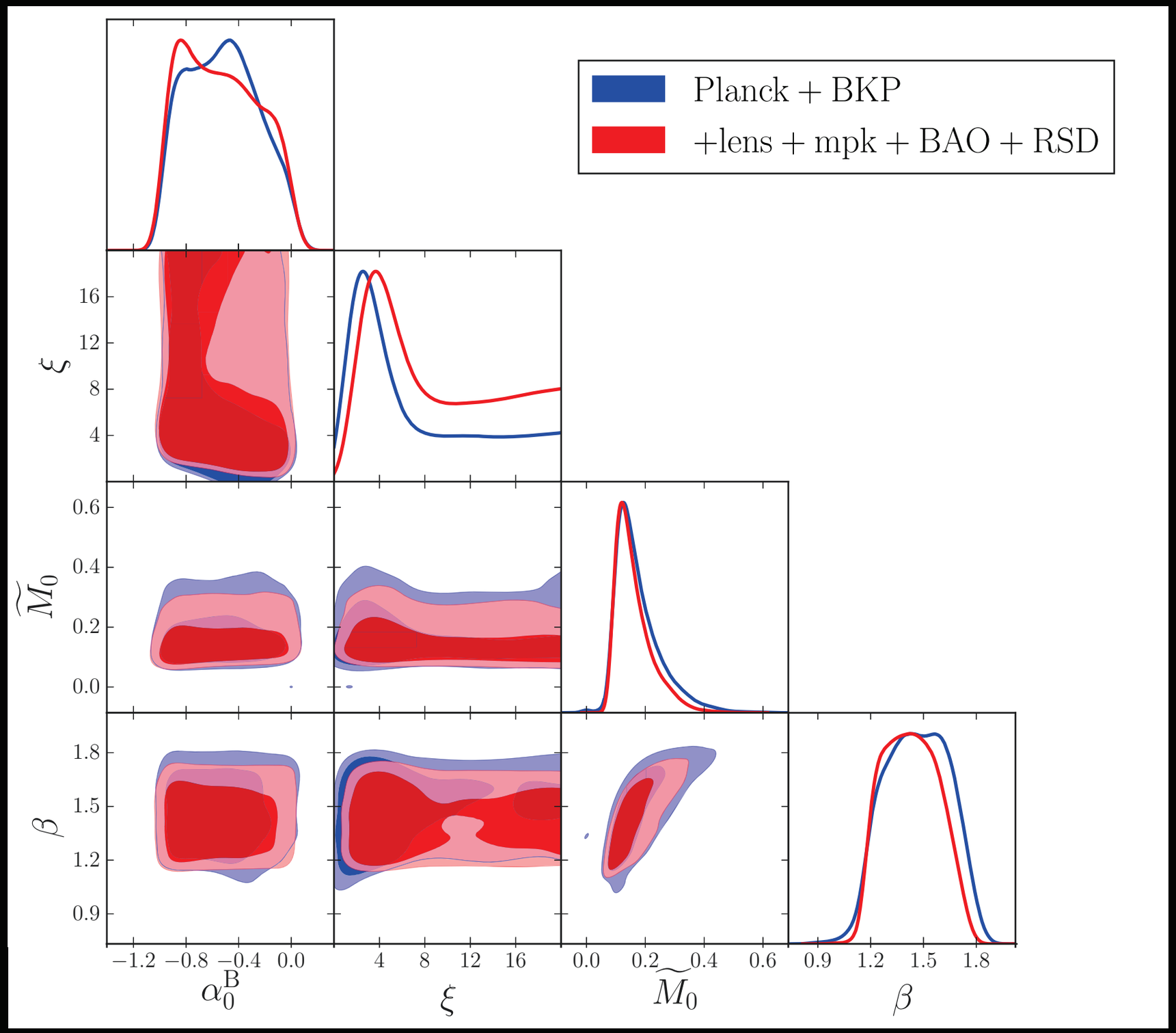
$$\alpha_B(z) = \alpha_0^B \, a^{\xi}$$

$$\frac{\delta M(z)}{M_{Pl}} = \tilde{M}_0 a^{\beta}$$

SDSS (galaxy survey) + Planck CMB + BOSS BAOs & RSDs + lensing data.

Kreisch & Komatsu, 1712.02710

THE CURRENT STATE OF PLAY



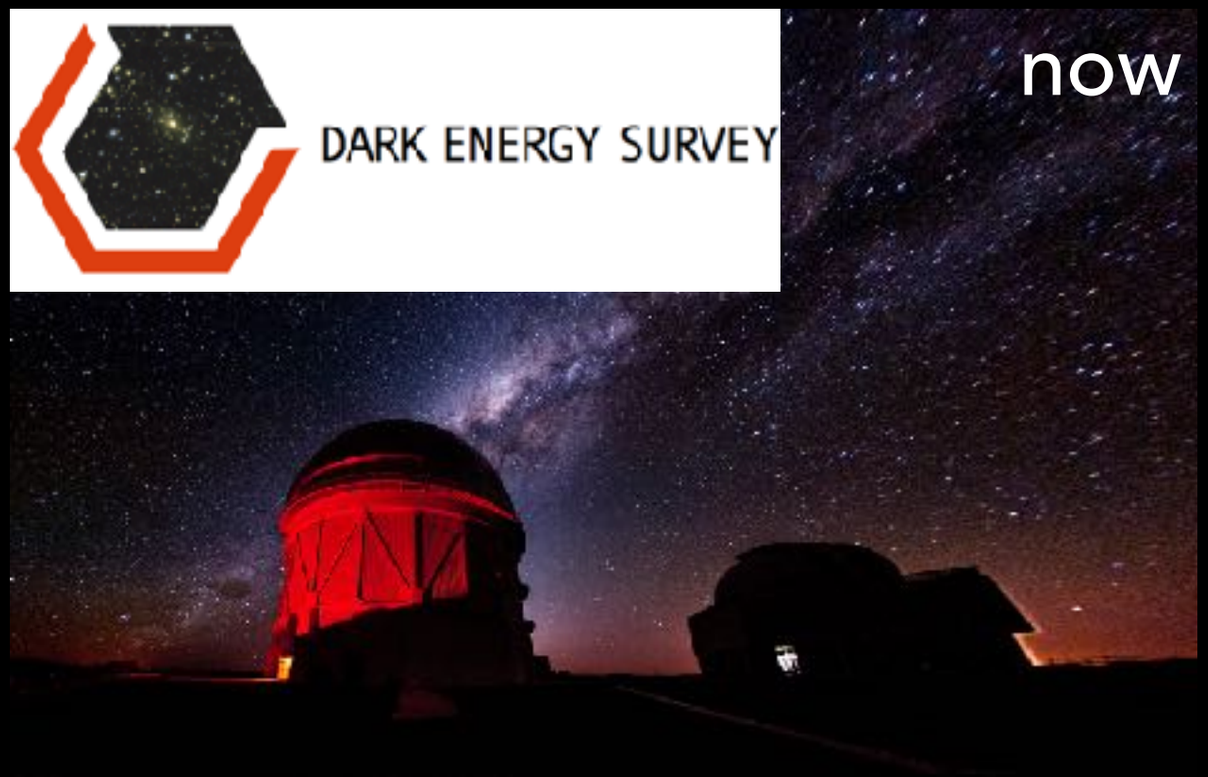
$$\frac{\delta M(z)}{M_{Pl}} = \tilde{M}_0 a^{\beta}$$

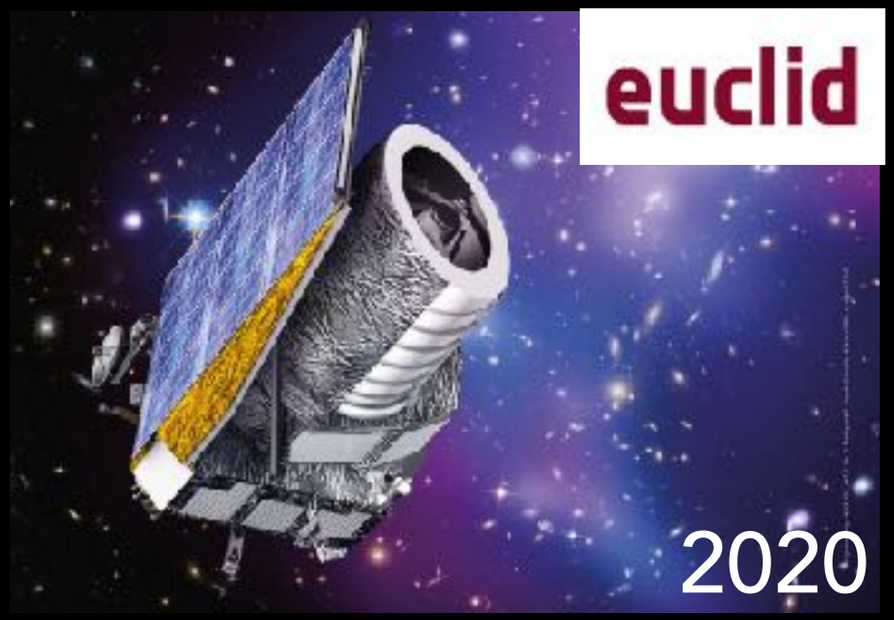
$$\alpha_B(z) = \alpha_0^B \, a^{\xi}$$

Caution: stability conditions lead to non-trivial contours.

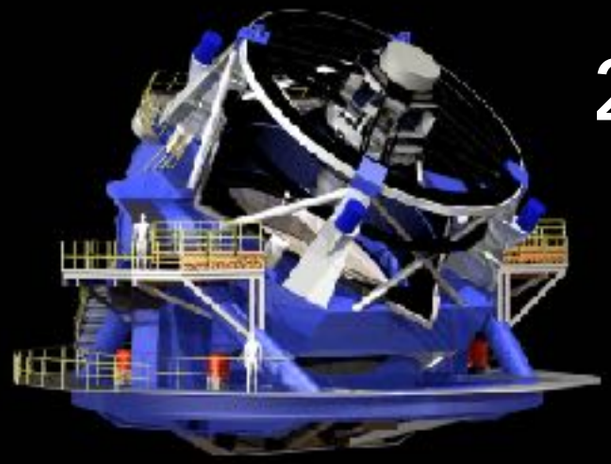
Kreisch & Komatsu, 1712.02710

ONGOING & FUTURE EXPERIMENTS





LSST
Large Synoptic Survey Telescope



2021

THE NEW THEORY LANDSCAPE

THE NEW THEORY LANDSCAPE

Quintessence

K-essence

Bigravity

Massive Gravity

Brans-Dicke

f(R) KGB

Cubic Galileon

(Beyond) Horndeski

Generalised Proca

Einstein-Aether

DHOST

Horava-Lifschitz

Quintic Galileons

Quartic Galileons

TeVeS

SVT

Fab Four

Uncertain? multiscalar-tensor, nonlocal gravity, Chaplygin gases.

THE NEW THEORY LANDSCAPE

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Cubic Galileon

Uncertain? multiscalar-tensor, nonlocal gravity, Chaplygin gases.

tessa.baker@physics.ox.ac.uk

CONCLUSIONS

1. The EFT of cosmological perturbations — agnostic & efficient tests of the gravity model landscape.

2. Framework can be linked directly to recent GW events with powerful results.

3. Goal: EFT used by next-generation experiments as standard format for dark energy constraints.

References:

1604.01396
1710.06394
1711.09893