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Using black hole formation to study thermalization

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Der Wissenschaftsfonds.

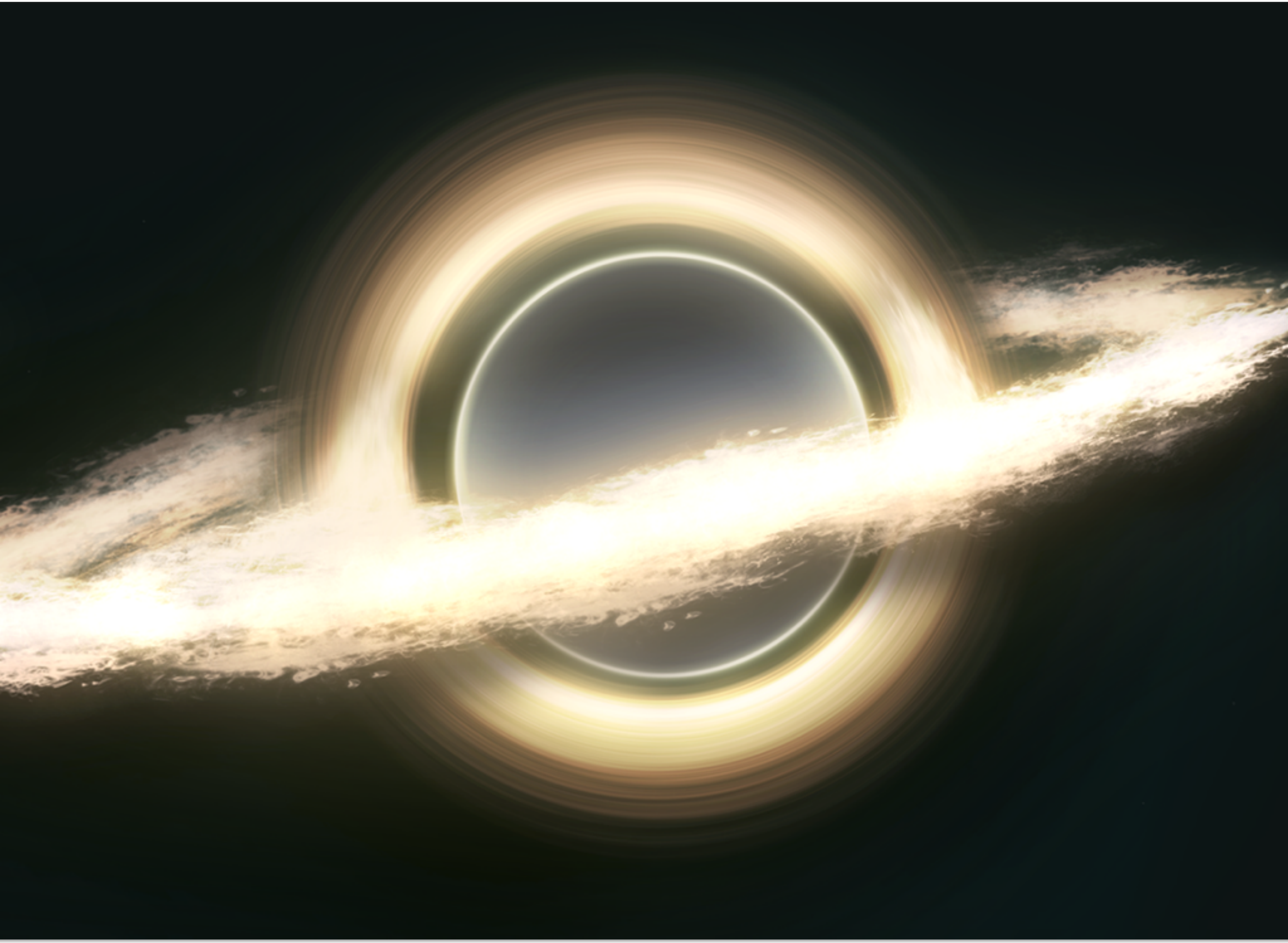
Outline

1. **Black holes & holographic principle**

2. **The AdS/CFT correspondence**

3. **Thermalization of the quark gluon plasma**

4. **Entaglement entropy**



Black holes

Classically: Simplest objects in the universe

- Mass
- Angular momentum
- Charge

Quantum mechanics: very complex objects

- Temperature, Hawking radiation
- Entropy: S ~ A
- Obey thermodynamic laws

(Bekenstein, Hawking, Unruh 70')

Black hole thermodynamics

Entropy

- Measure for disorder
- Information about a system what we don't know: counts micro states

Classical example: bathtub

- Macroscopically: Temperature, Volume
- Microscopically: number, position and momentum of water molecules
- Entropy: $S = k \log W \approx N \propto V$

Black hole thermodynamics

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Black holes

- Entropy: $S_{BH} = \dfrac{A}{4G_N}$
- Holographic objects

('t Hooft 1993, Susskind 1995)

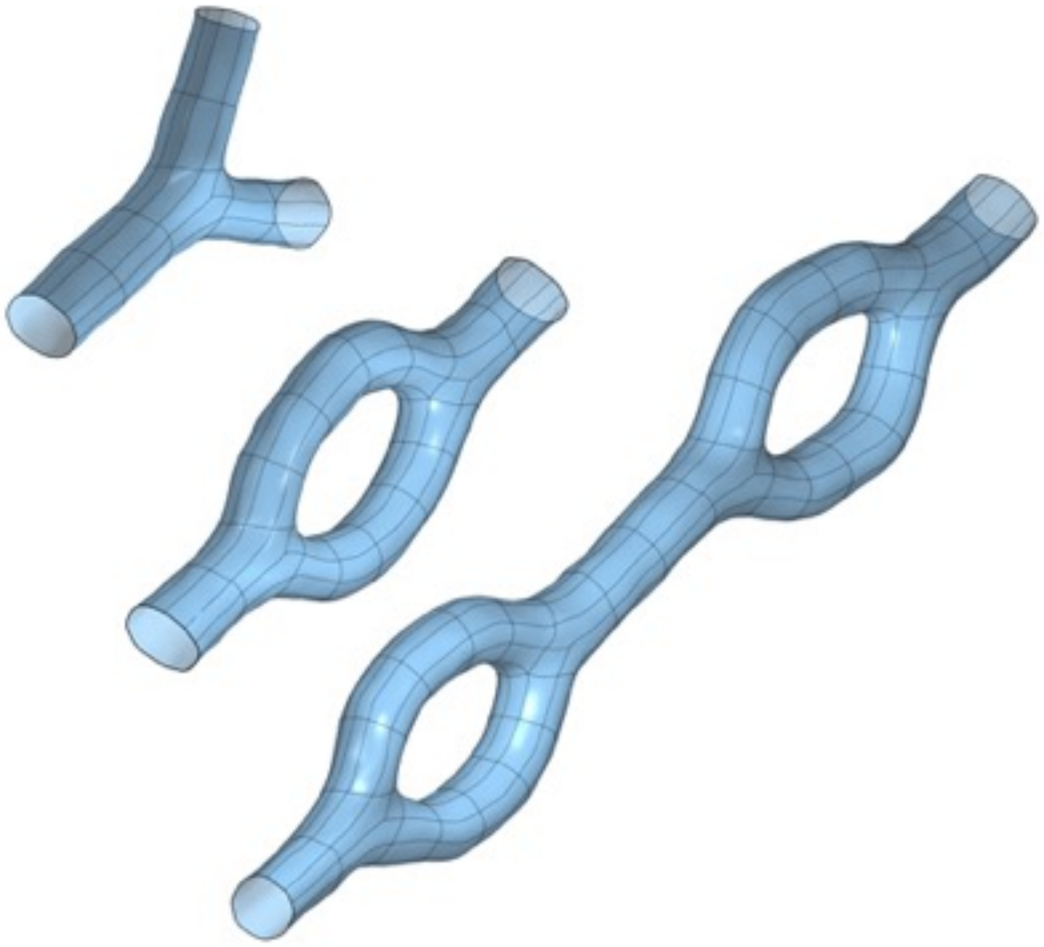
The holographic principle

A true theory of quantum gravity must live in one fewer dimension

What are the micro states?

String theory

- Fundamental objects are one-dim strings
- Supersymmetric theory
- Requires 10 dim to be consistent
- Low energy limit: Supergravity



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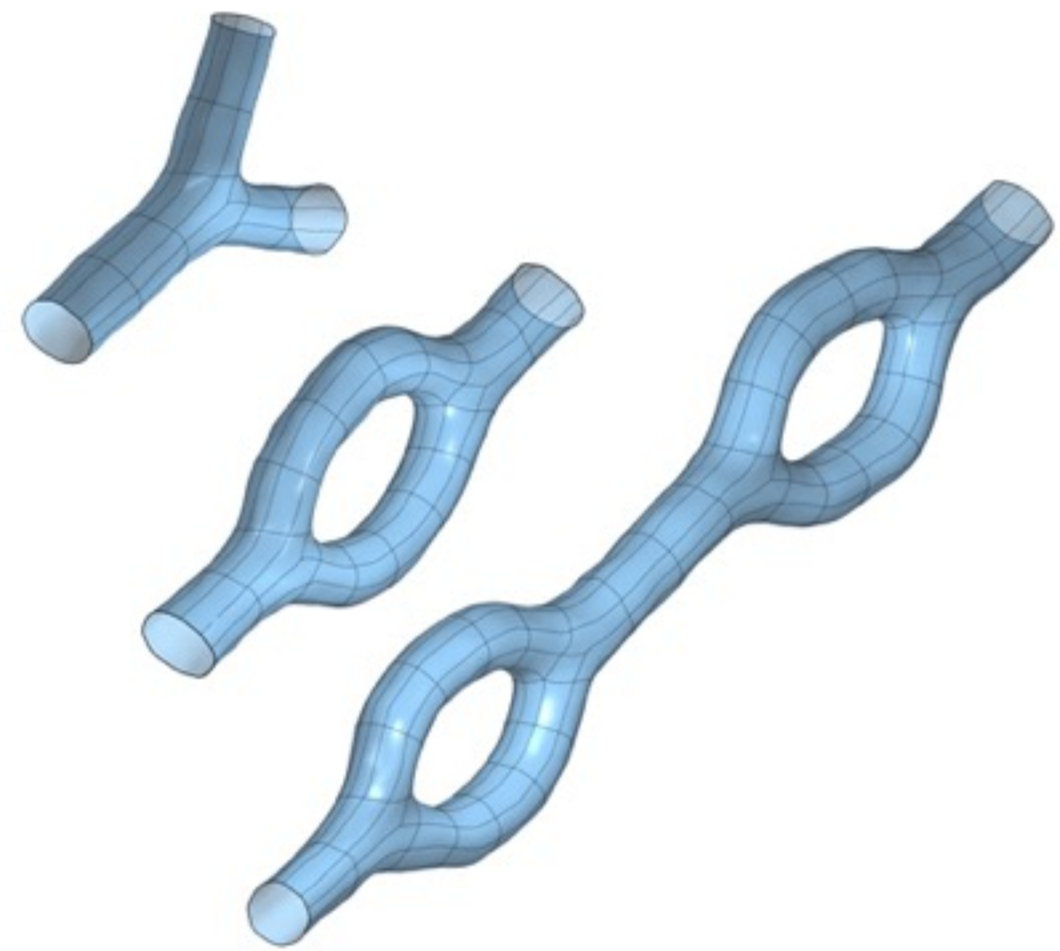


Black hole entropy

(Strominger, Vafa 1995)

From string theory it follows:

$$S_{BH} = \frac{A}{4G_N}$$

The gauge/gravity duality

Realisation of the holographic principle

Type IIB string theory on $AdS_5 \times S^5$ is dual to $N = 4$ supersymmetric Yang Mills theory living one the boundary of AdS_5

(Maldacena 1997)

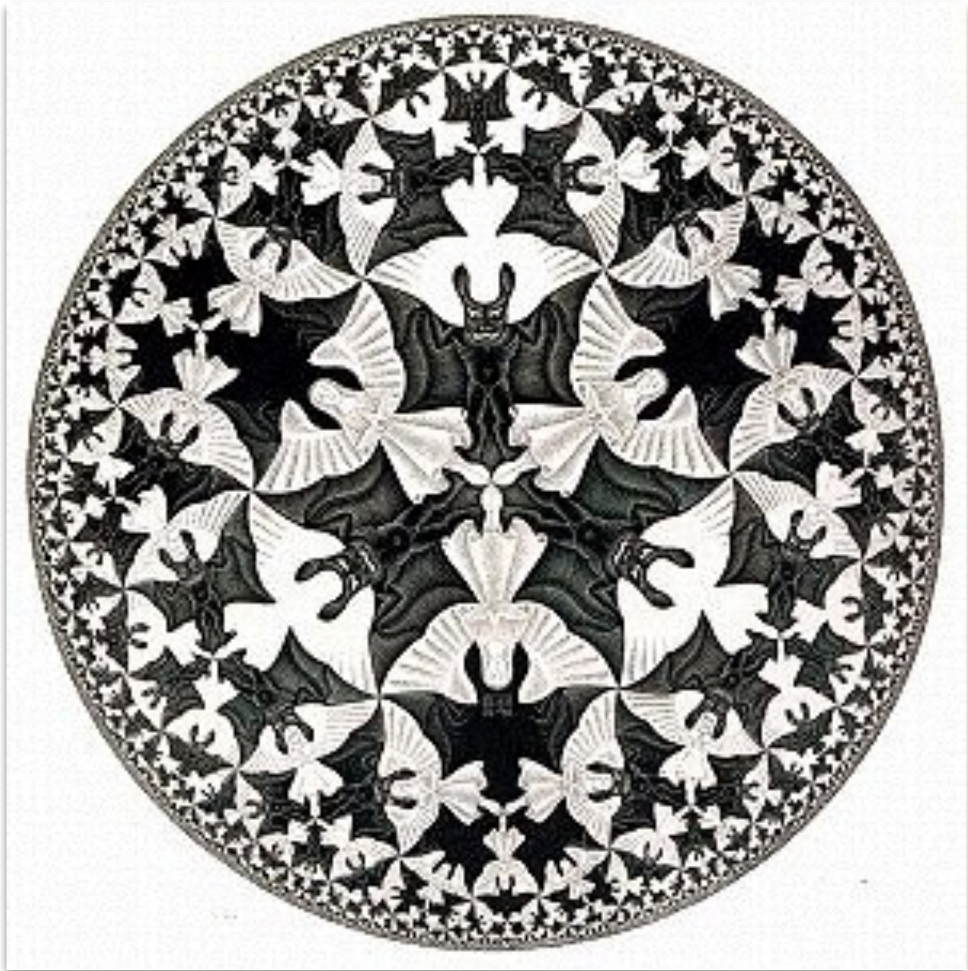
The gauge/gravity duality

Realisation of the holographic principle

Type IIB string theory on **AdS**$_5$ x S^5 is dual to N = 4 supersymmetric Yang Mills theory living one the boundary of AdS_5

Anti-deSitter space

- Maximal symmetric space with negative curvature: hyperbolic space
- Solution to Einsteins equation with negative cosmological constant
- It has a boundary



M.C. Escher, Haven & Hell

The gauge/gravity duality

Realisation of the holographic principle

Type IIB string theory on $AdS_5 \times S^5$ is **dual** to $N = 4$ supersymmetric Yang Mills theory living one the boundary of AdS_5

Duality

weak coupling $\longleftarrow$ dictionary $\longrightarrow$ **strong coupling**

- Supergravity
- Perturb. SYM theory

- strongly coupled SYM
- Quantum gravity

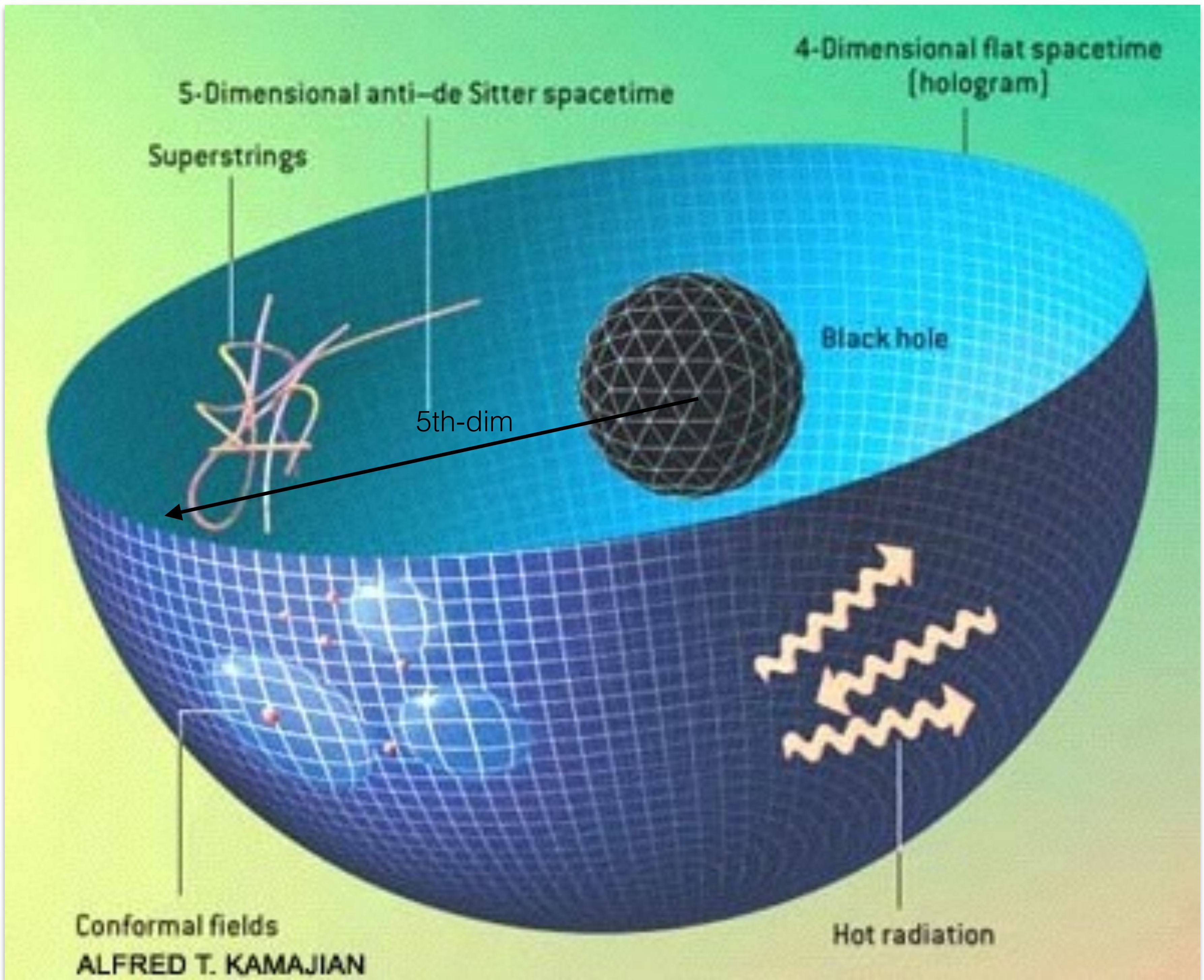
$$\frac{L^4}{l_s^4} = 4\pi g_s N = g_{YM}^2 N = \lambda$$

The gauge/gravity duality

The power of duality

opens a new way to study theories in the hardly accessible regime
via the dual description

Classical gravity on AdS_5 is dual to a strongly coupled field theory living on the boundary



Strongly coupled systems

Condensed matter systems

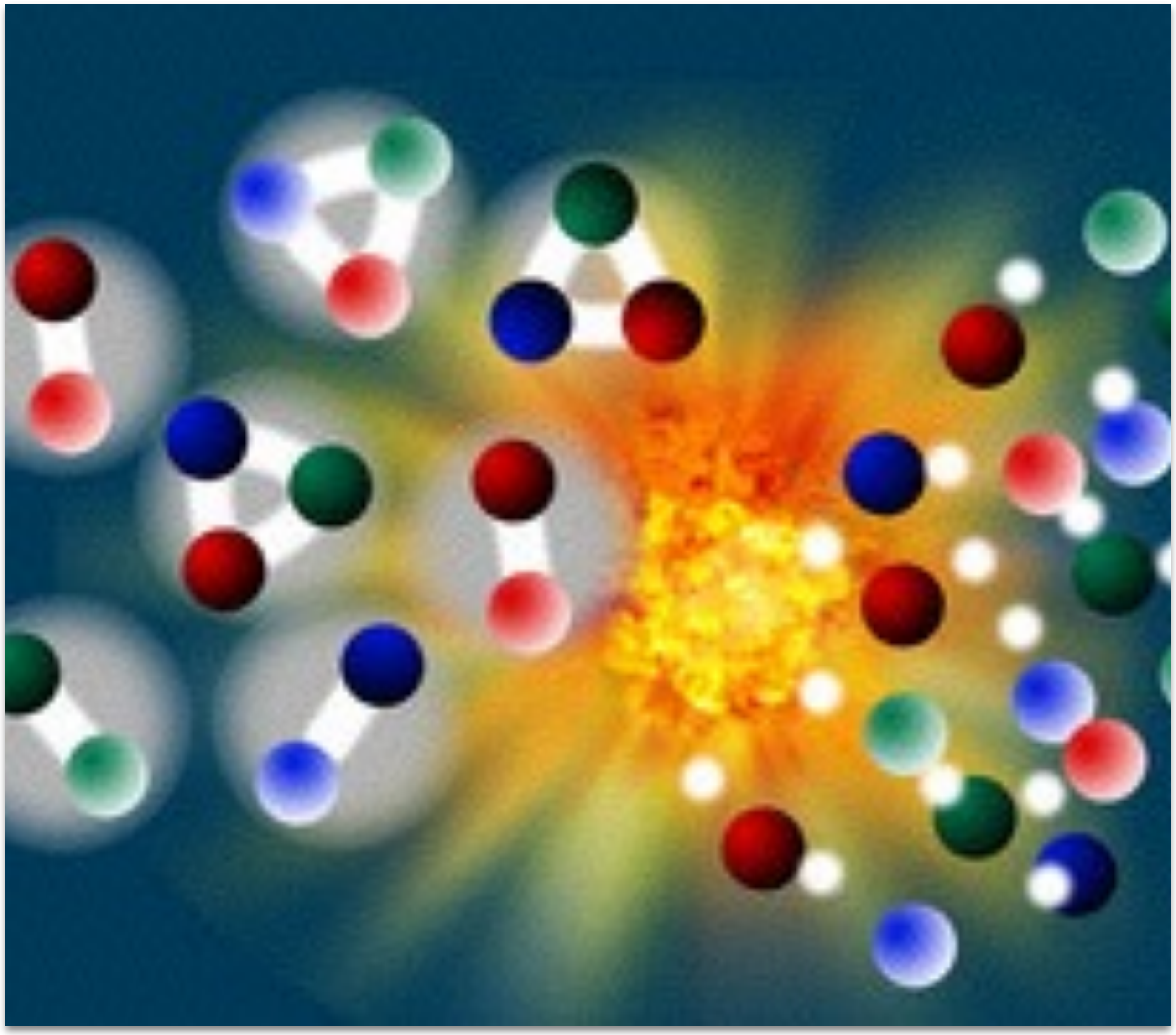
- Graphene
- High temperature superconductors
- Strange metals
- Non fermi liquids

Quantum Chromodynamics

- Theory of strong interactions
- Confinement
- Asymptotic freedom
- At hight temperature: phase transitions (cross over) to Quark Gluon Plasma

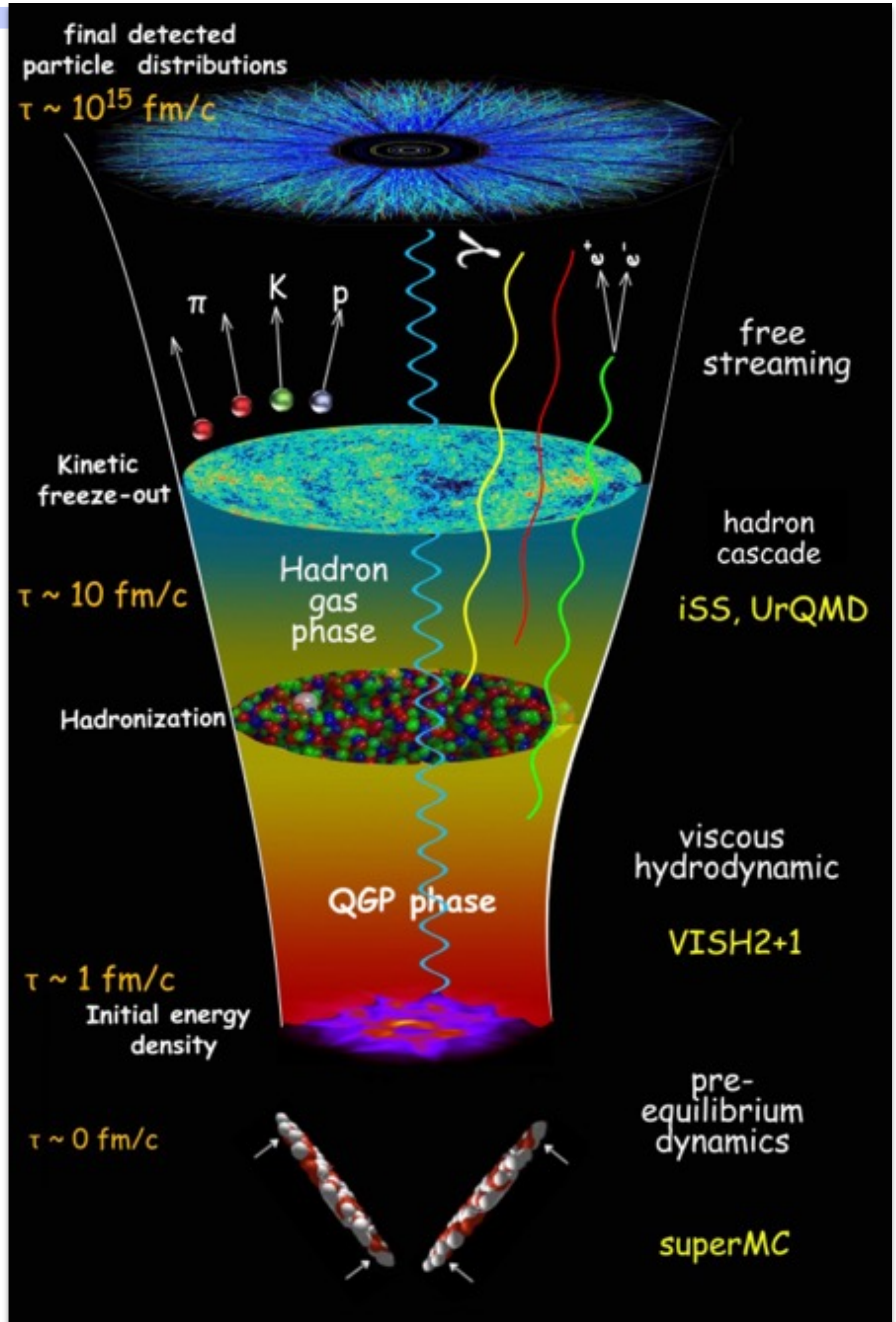
Quark Gluon Plasma

- Created in heavy ion collisions at RHIC and the LHC
- Temp. ~ 10^{12} K
- Lifetime. ~ 10^{-23} sec
- Size ~ 10^{-14} m
- Behaves as a strongly coupled fluid
- Rapid thermalization
- Thermalization process not well understood



Stages of a heavy ion collision

- Nontrivial observation: hydro description of fireball evolution works extremely well
- Surprise from RHIC/LHC: Extremely fast equilibration into almost ideal fluid behaviour — hard to explain via weakly coupled quasiparticle picture
- How does the transition to viscous hydro happen?



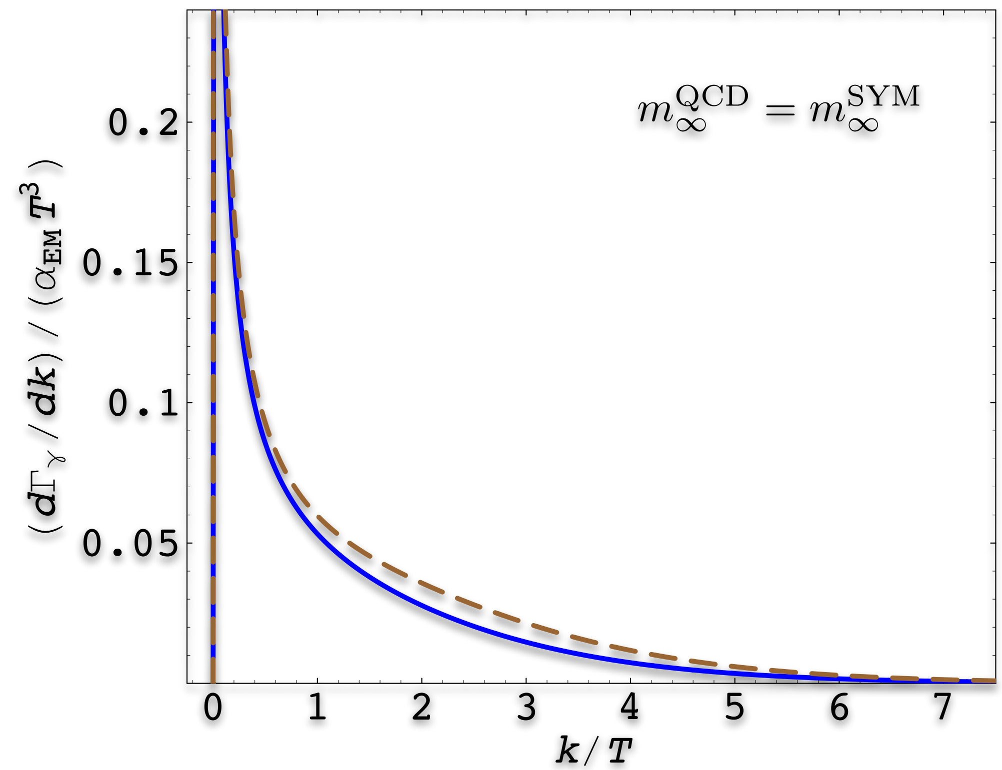
Strategy

Use SYM theory where the strongly coupled regime is accessible

Strategy

Similarities to QCD at finite T

- SUSY is broken
- No confinement above T_c and chiral condensate melts away
- QCD has a conformal window
- Photon emission rates are similar at weak coupling



Photon emission *(Huot et al 2006)*

Advantage

- Both weak and strong coupling can be studied analytically
- Transport coefficients and time dependent quantities can be computed

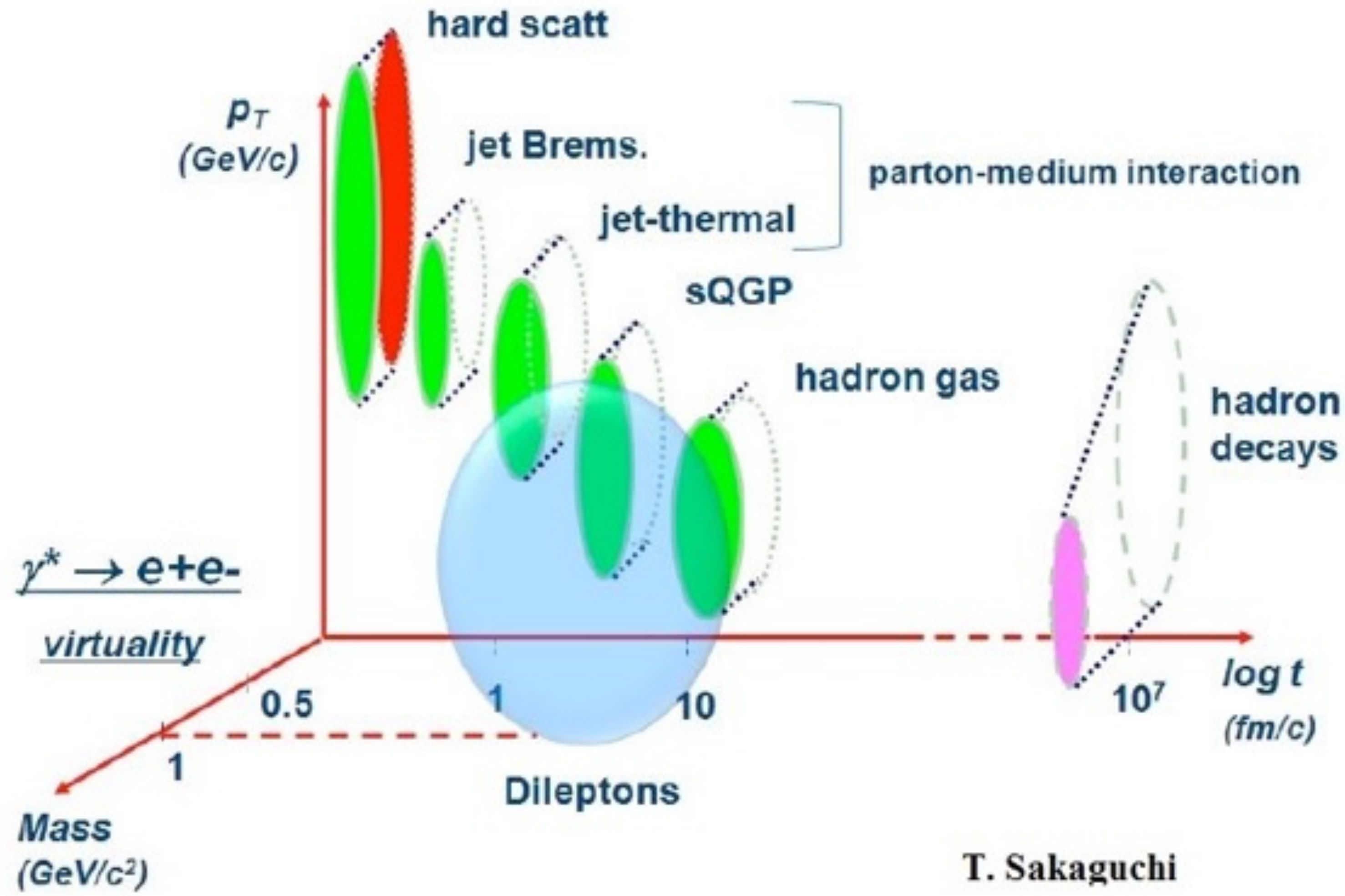
Probing the plasma

Quantities of interest

- Stress energy tensor
- Transport coefficients
- Spectral function:

$$\chi^{\mu}_{\mu} = -2\mathrm{Im}(\Pi^{\mathrm{ret}})^{\mu}_{\mu}(k_0)$$

- Photon production rates

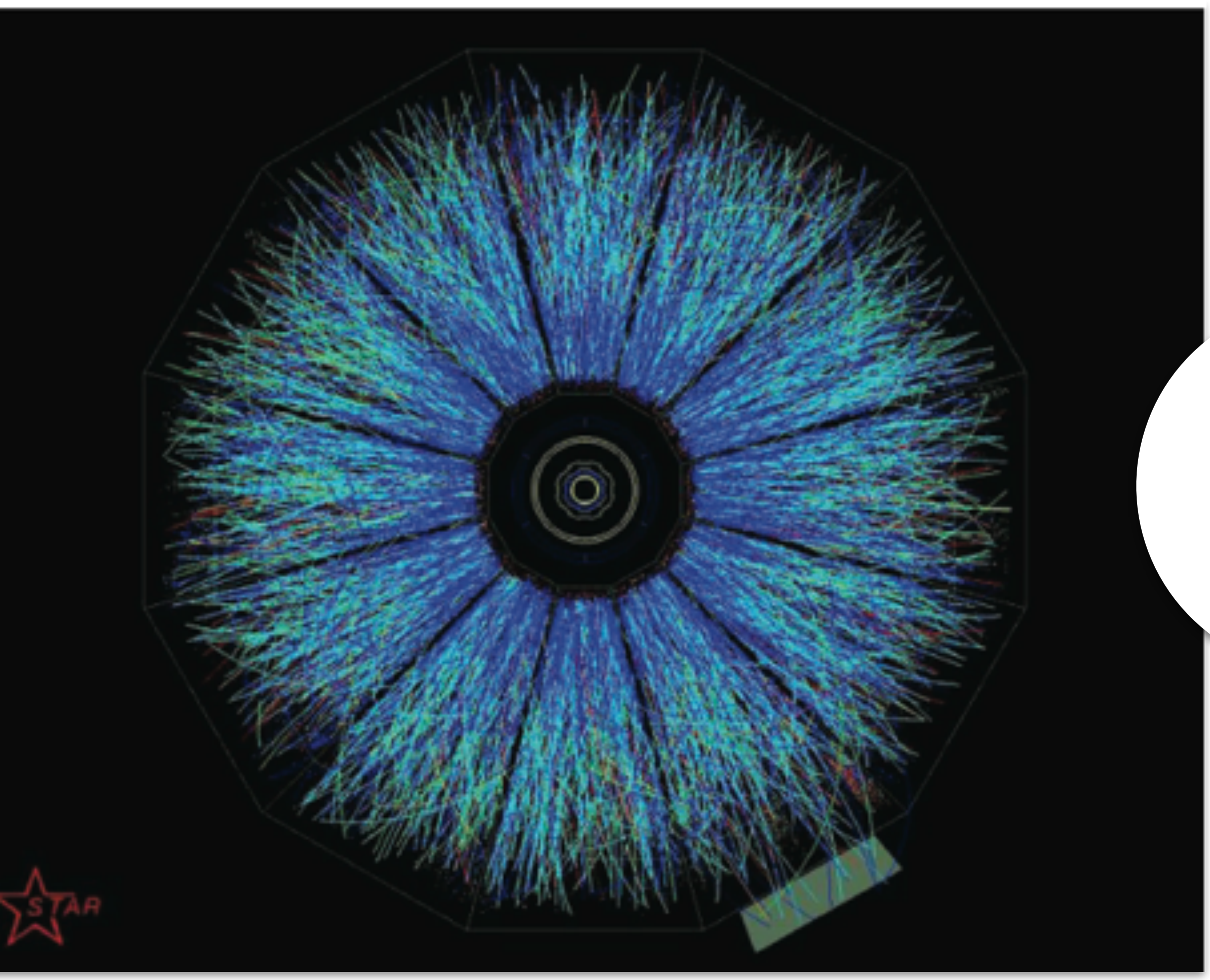


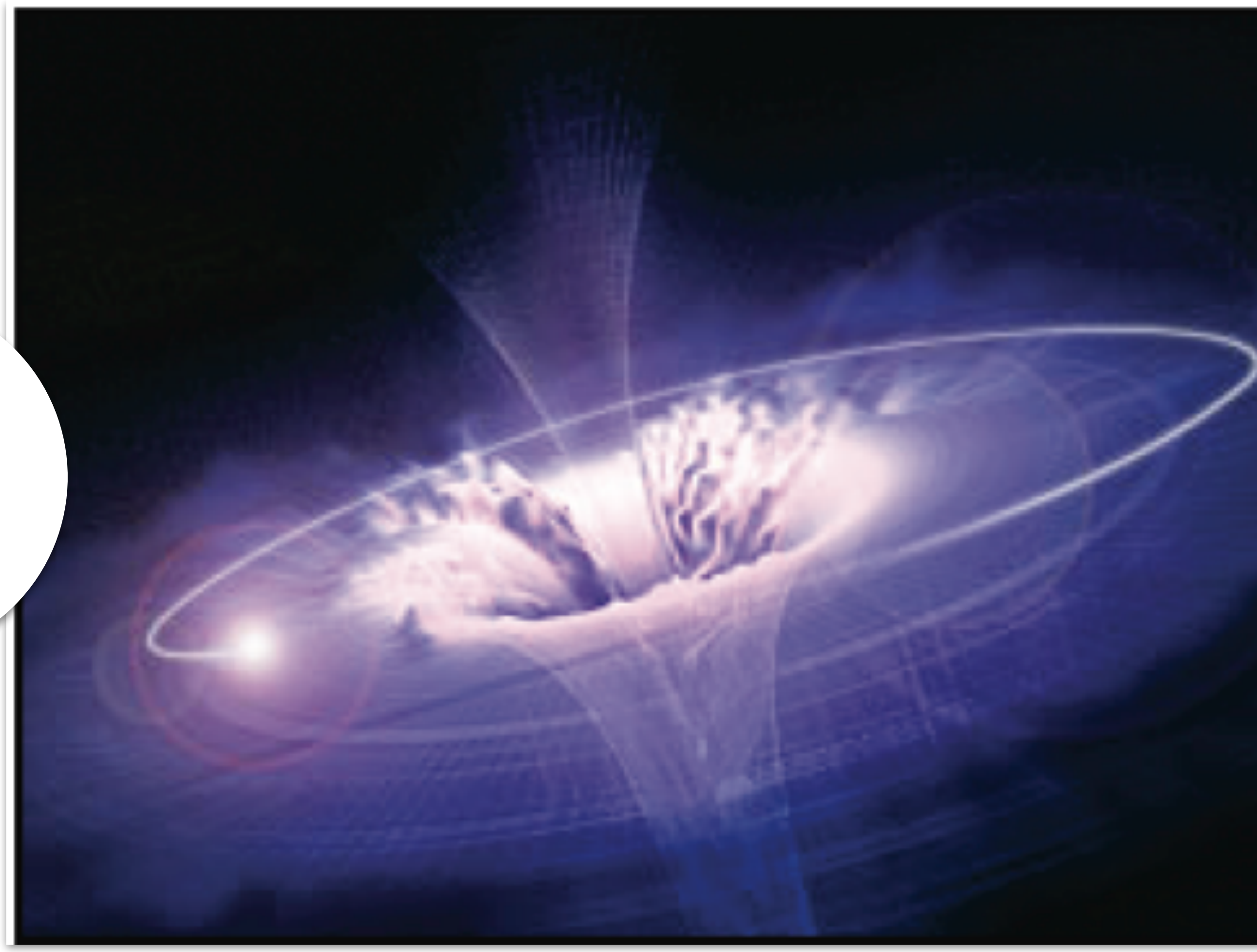
Photons as probe

- Emitted at all stages of the collision
- Once produced they stream through the plasma almost unaltered
- Provide observational window into the thermalization process

Strongly coupled thermalization

Thermalization | **Black hole formation**





Thermalization process of strongly coupled SYM theory is mapped to black hole formation in asymptotically anti-de Sitter space

Shear Viscosity from black holes

Shear viscosity for strongly coupled N = 4 SYM plasma from black holes

- small
- universal
- difficult to compute by other means
- close to experimental value

shear viscosity

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

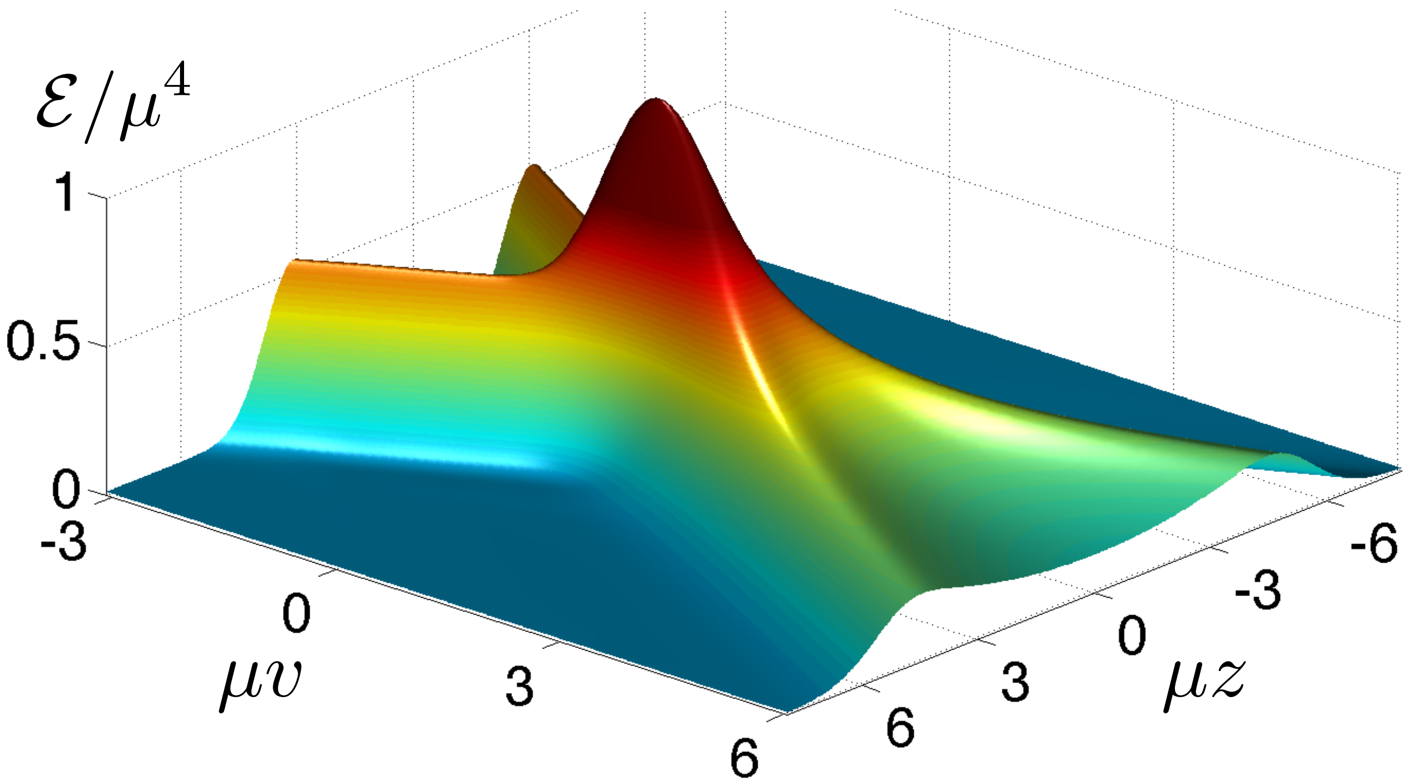
entropy density

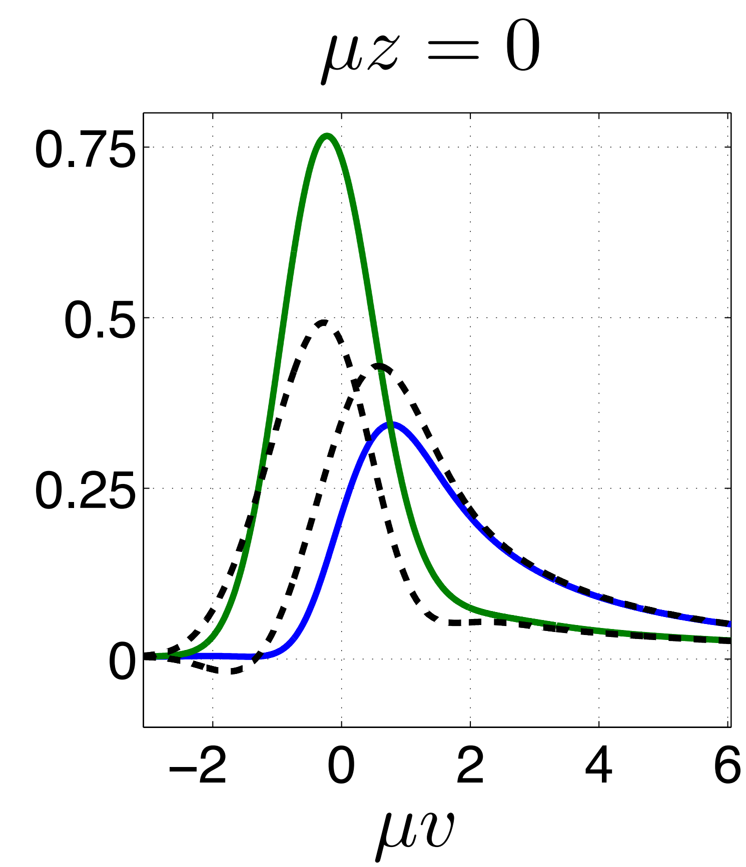
Policastro, Son, Starinets (2001)

Thermalization at strong coupling

Colliding shocks in AdS

$$\langle T_{\mu\nu} \rangle = \frac{N_c^2}{2\pi^2} \mathrm{diag}[\epsilon, P_{\parallel}(t), P_{\perp}(t), P_{\perp}(t)]$$





Lessons from gauge/gravity duality

- Thermalization time naturally short t_{eq}~1/T
- Hydrodynamization ≠ thermalization, isotropization

Chesler & Yaffe

Thermalization processes

Bottom up scenario

- Weak coupling
- Scattering processes

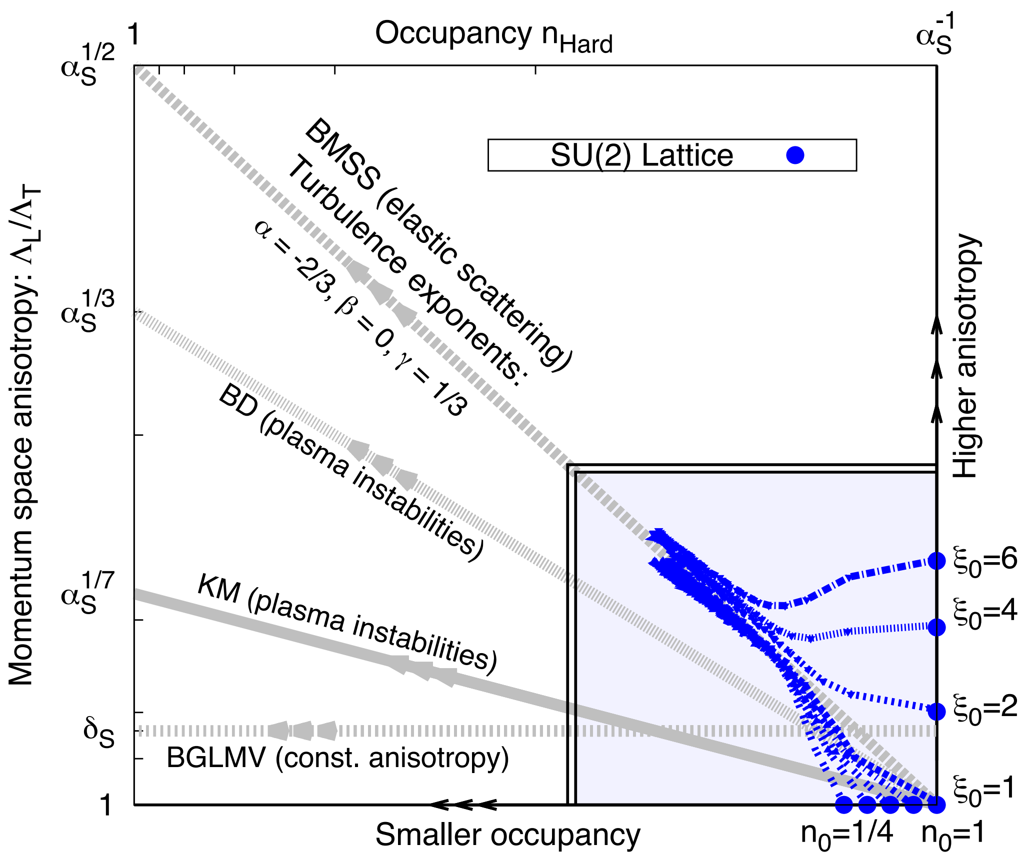
  In the early stages many soft gluons are emitted which then thermalise the system (*Baier et al (2001)*)

- Driven by instabilities

  Instabilities isotropize the momentum distributions more rapidly than scattering processes
  (*Kurkela, Moore (2011)*)

Top down scenario

- Strong coupling
- UV modes thermals first



(*Berges et al.*)

Quasinormal modes

- Characterize the response of the system to inf. perturbations
- Dispersion relation of field excitations

$$\omega_n(q) = M_n(q) - i\Gamma_n(q),$$

- Reveal a striking difference between weak and strong coupling

Weak coupling

- Long lived quasiparticles:

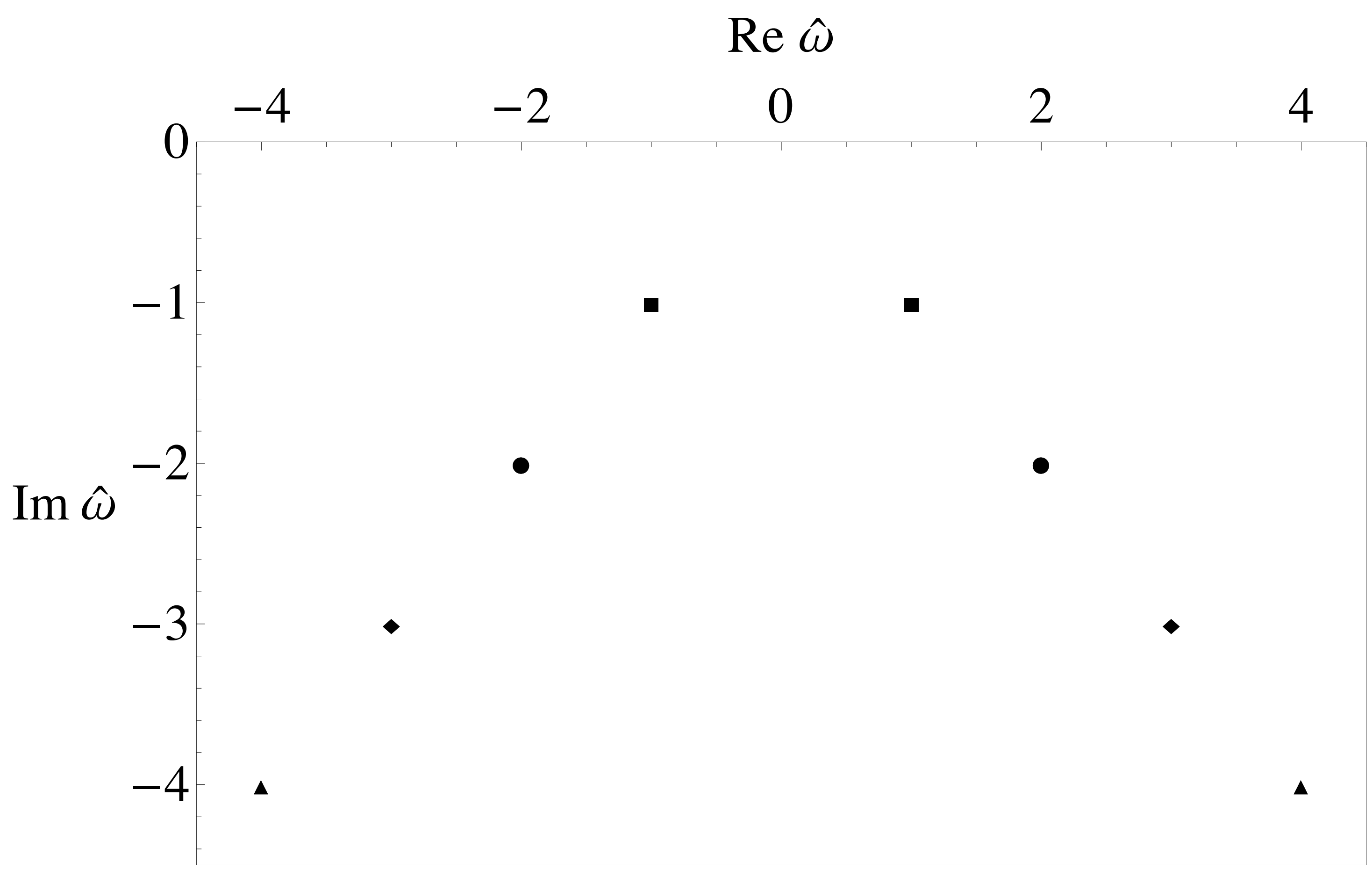
  $\mathrm{Im}(\omega_n) \ll \mathrm{Re}(\omega_n)$

- Bottom up thermalization *(Baier et al 2001)*

Strong coupling

- No quasiparticle picture, infinite tower QNM
- Thermalization always top down

QNM at infinite coupling

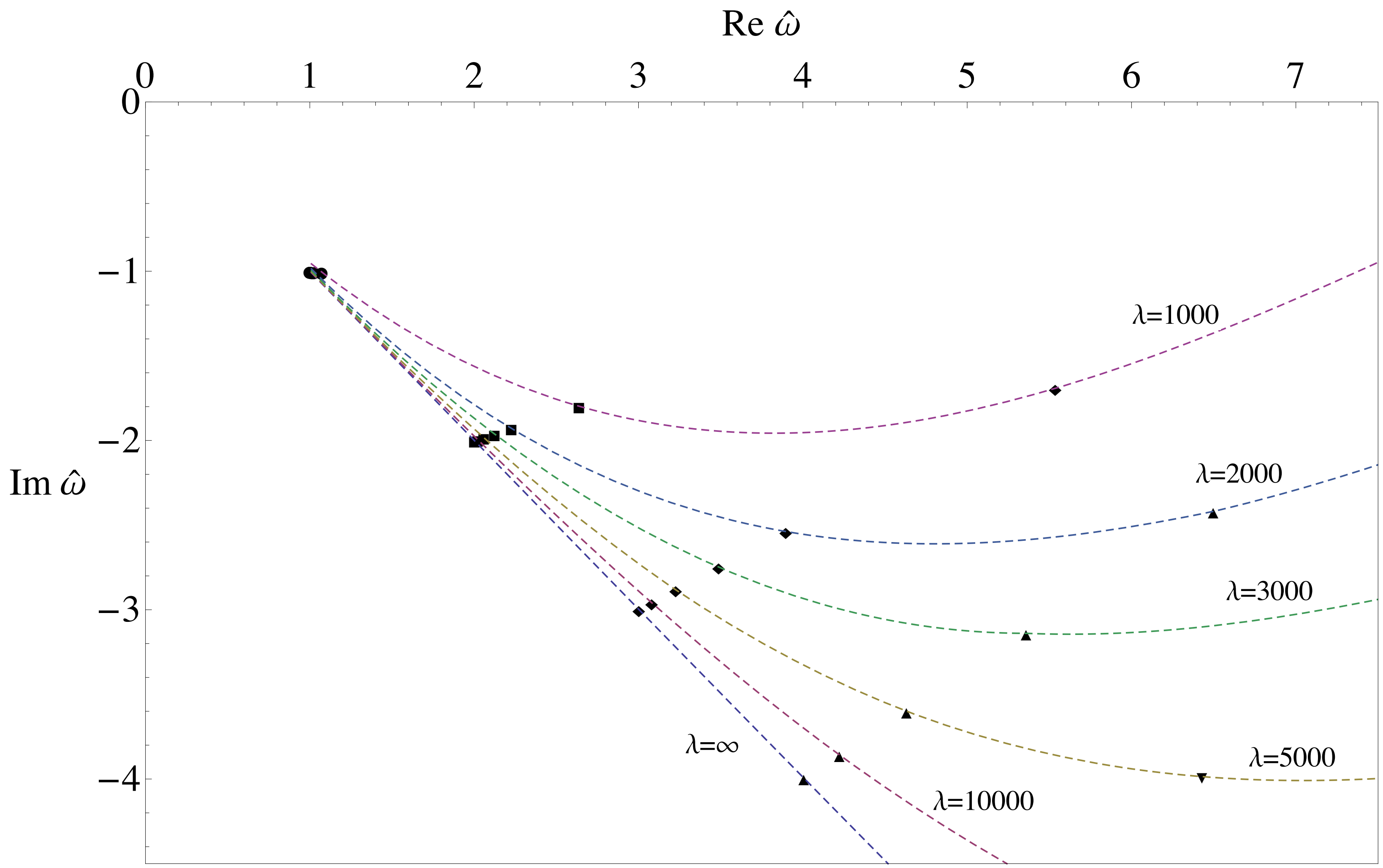


$$\omega_n|_{q=0} = n(\pm 1 - i)$$

(Kovtun, Starinets)

- Pole structure of EM current-current correlator displays usual quasinormal mode spectrum at infinite coupling
- How does the QNM spectrum get modified at finite coupling?

QNM at finite coupling



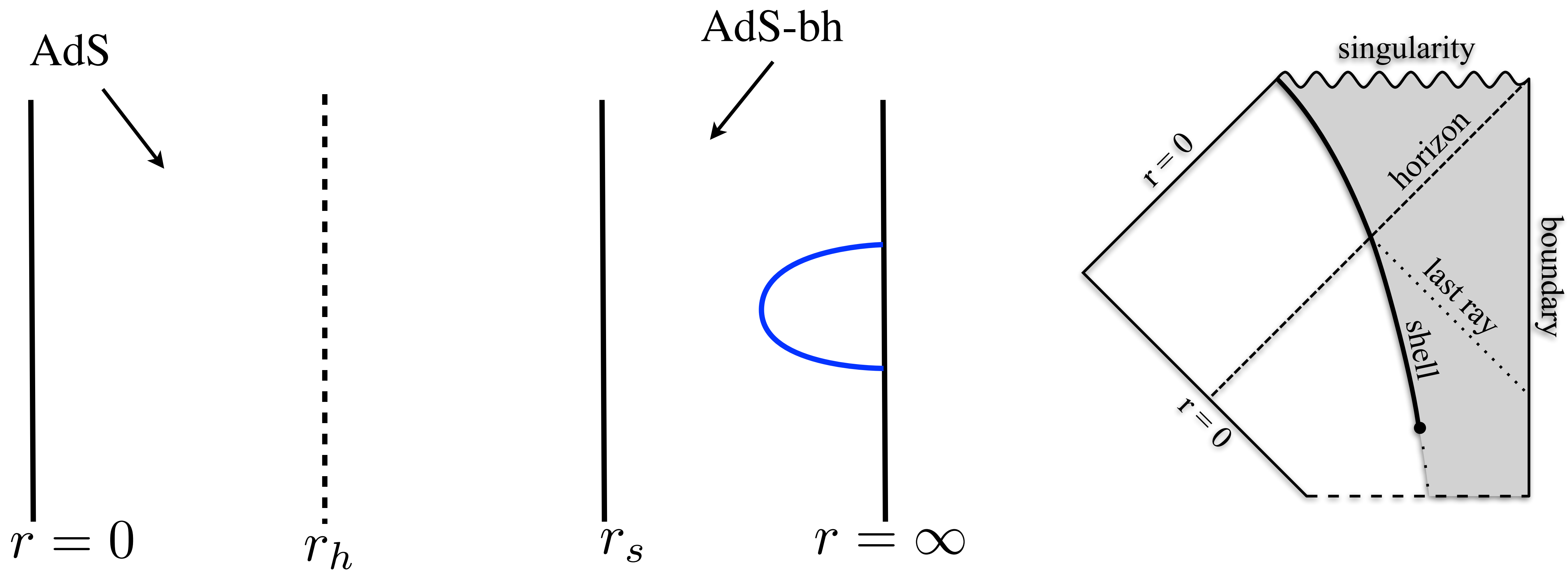
Steineder, SS, Vuorinen (2013), SS (2014)

- Effect of decreasing coupling: Imaginary part increases, lowering the decay rate of the excitations ⇒ modes become longer - lived
- Larger impact on higher energetic modes
- Convergence of strong coupling expansion not guaranteed when shift is of $\mathcal{O}(1)$

Far from equilibrium dynamics

Danielsson, Keski-Vakkuri, Kruczenski (1999)

The falling shell setup



Thermalization from geometric probes:

- Entanglement entropy and Wilson loop: always top down thermalization

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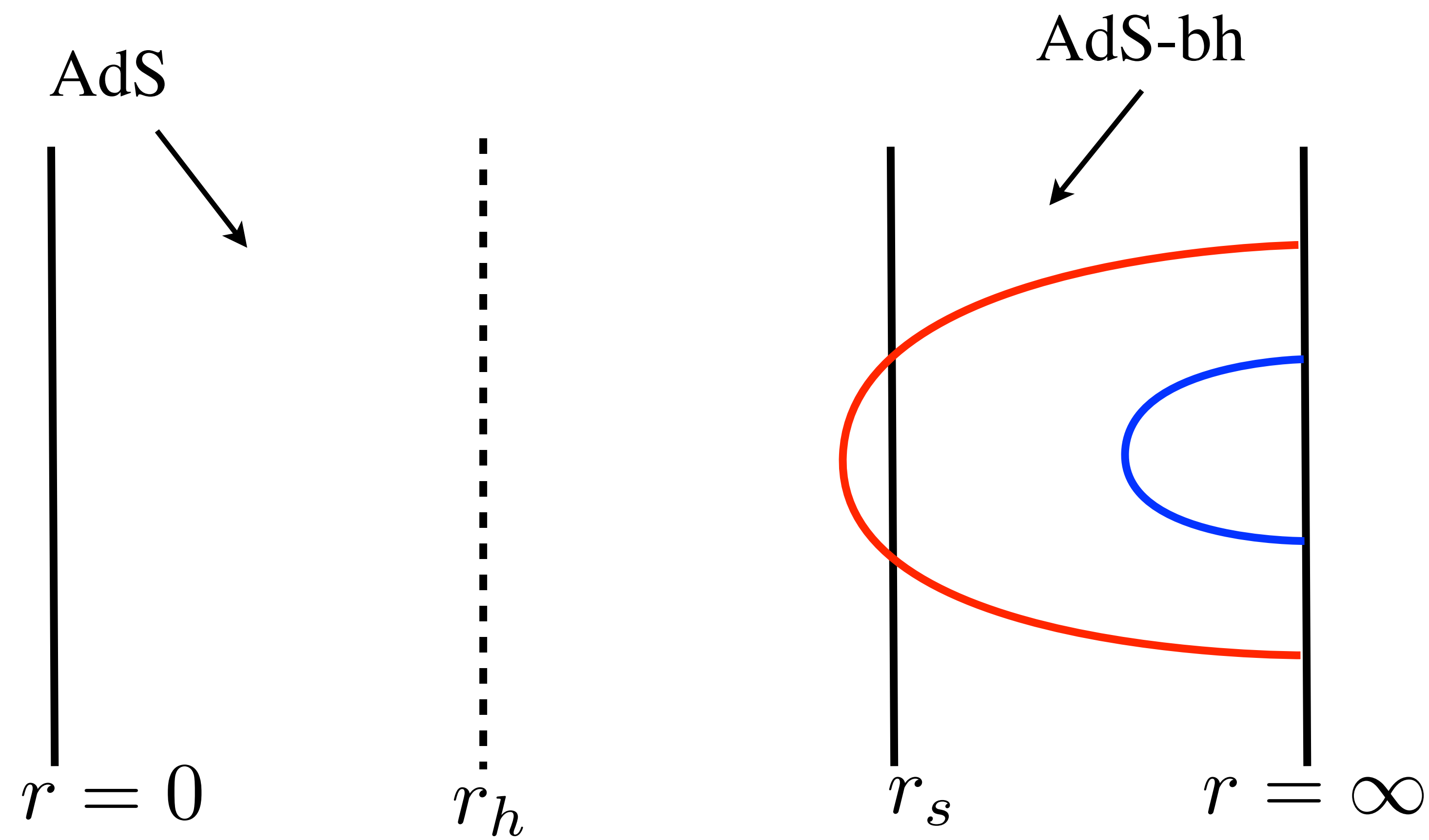

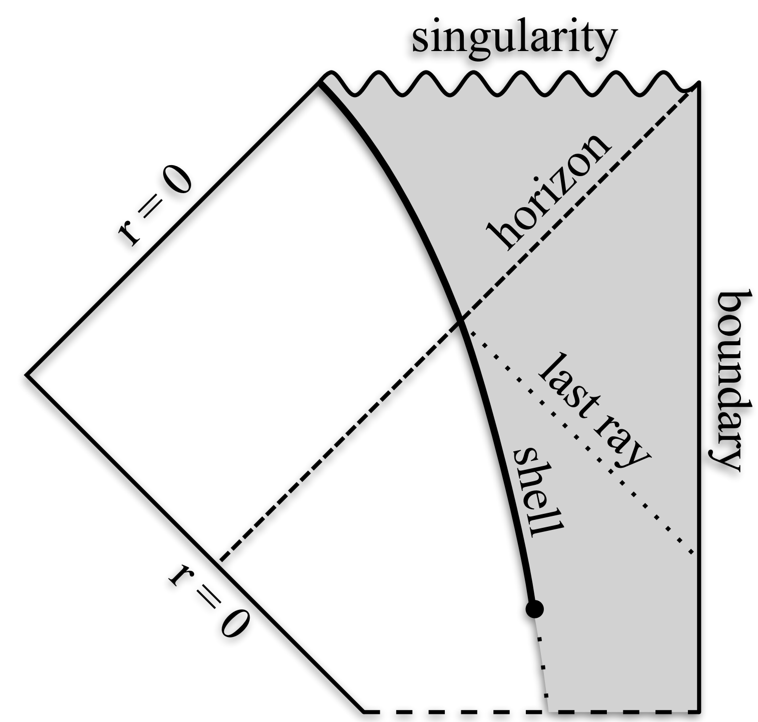

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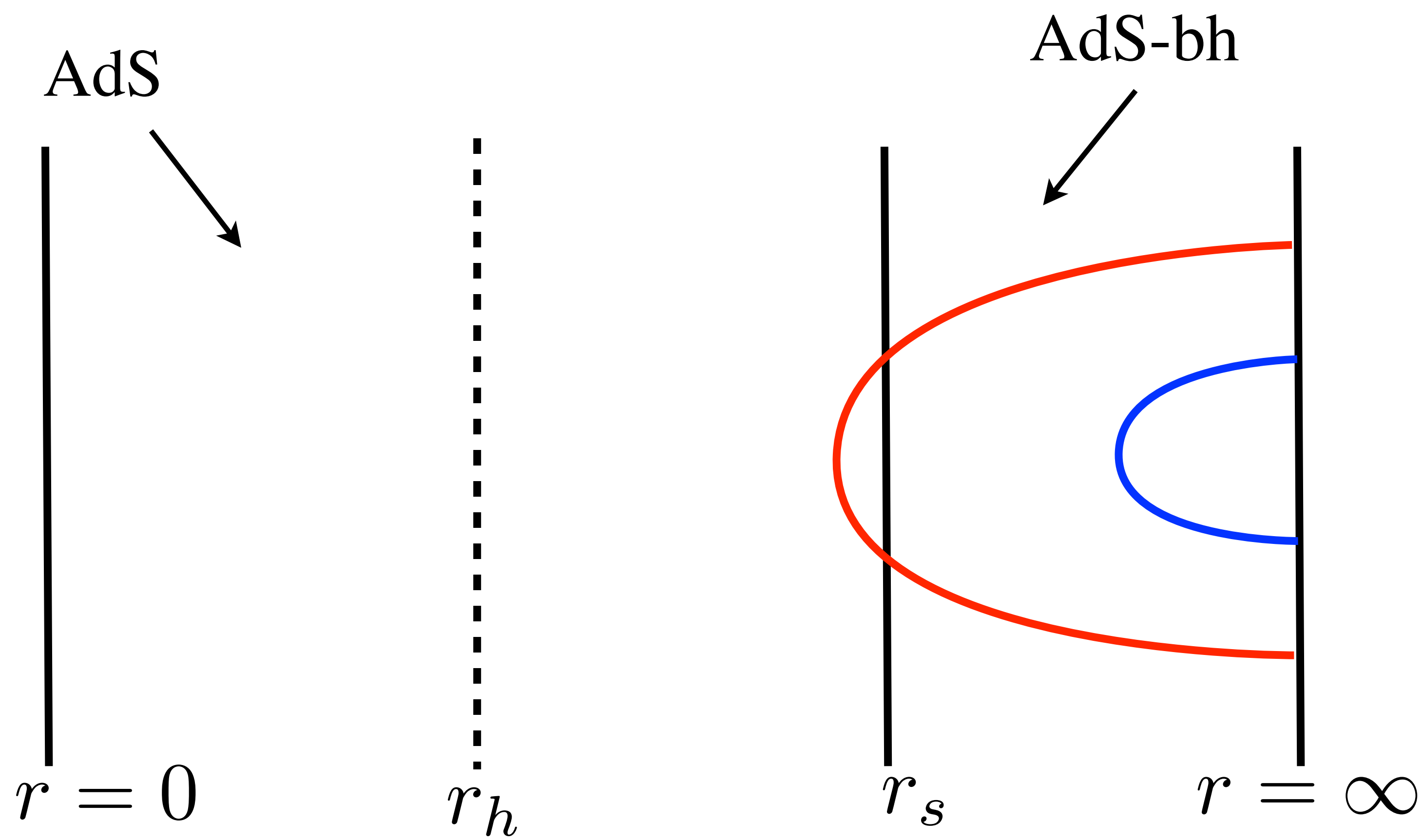
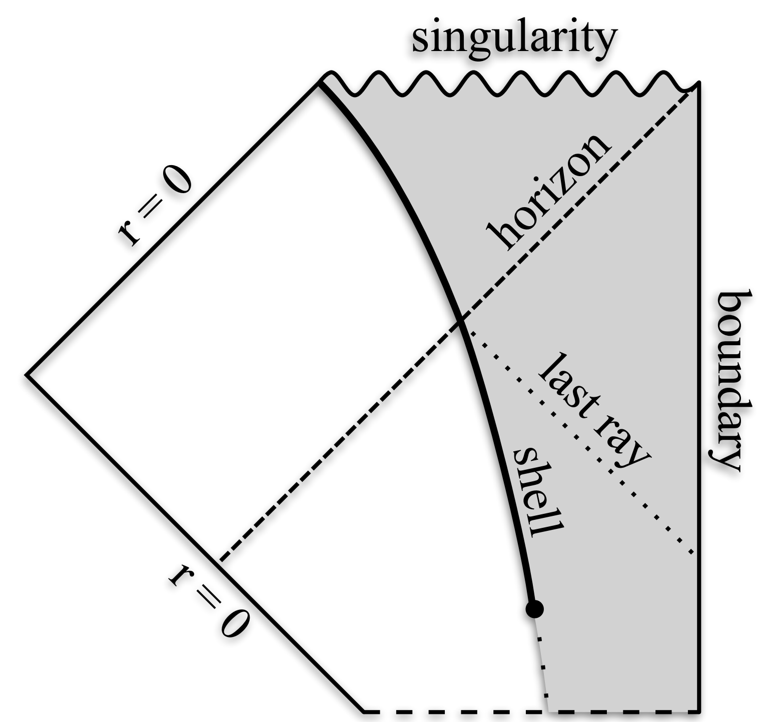




Quasistatic approximation

- Energy scale of interest >> characteristic time scale of shells motion

Photon spectral density

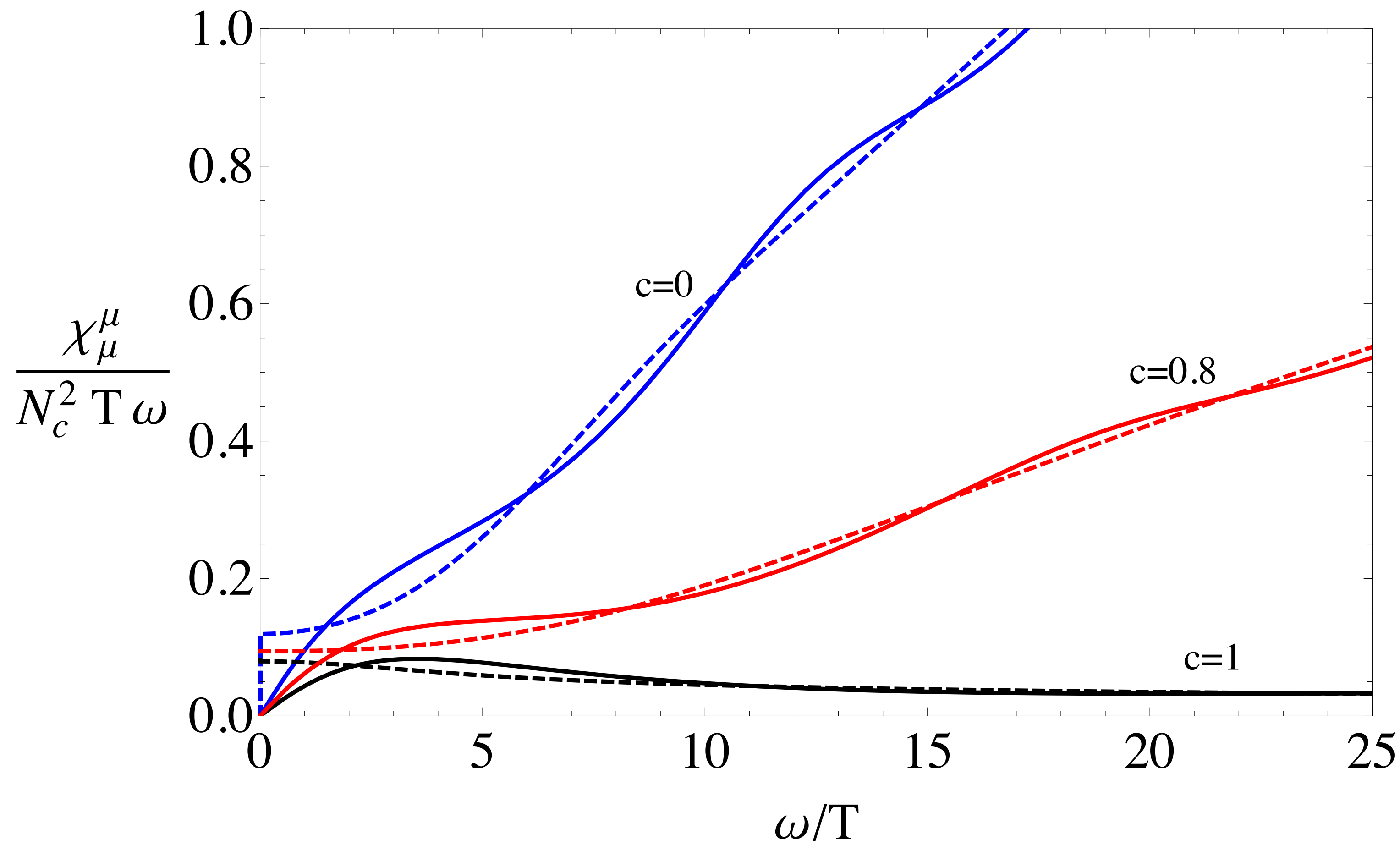
Baier, Taanila, SS, Vuorinen (2012)

natural quantity to study: **spectral density**: $\chi^{\mu}_{\mu} = -2\mathrm{Im}(\Pi^{\mathrm{ret}})^{\mu}_{\mu}(k_0)$

- virtuality

$$v = \frac{\hat{\omega}^2 - \hat{q}^2}{\hat{\omega}^2}$$

- parametrize $\quad q = c\,\hat{\omega}$



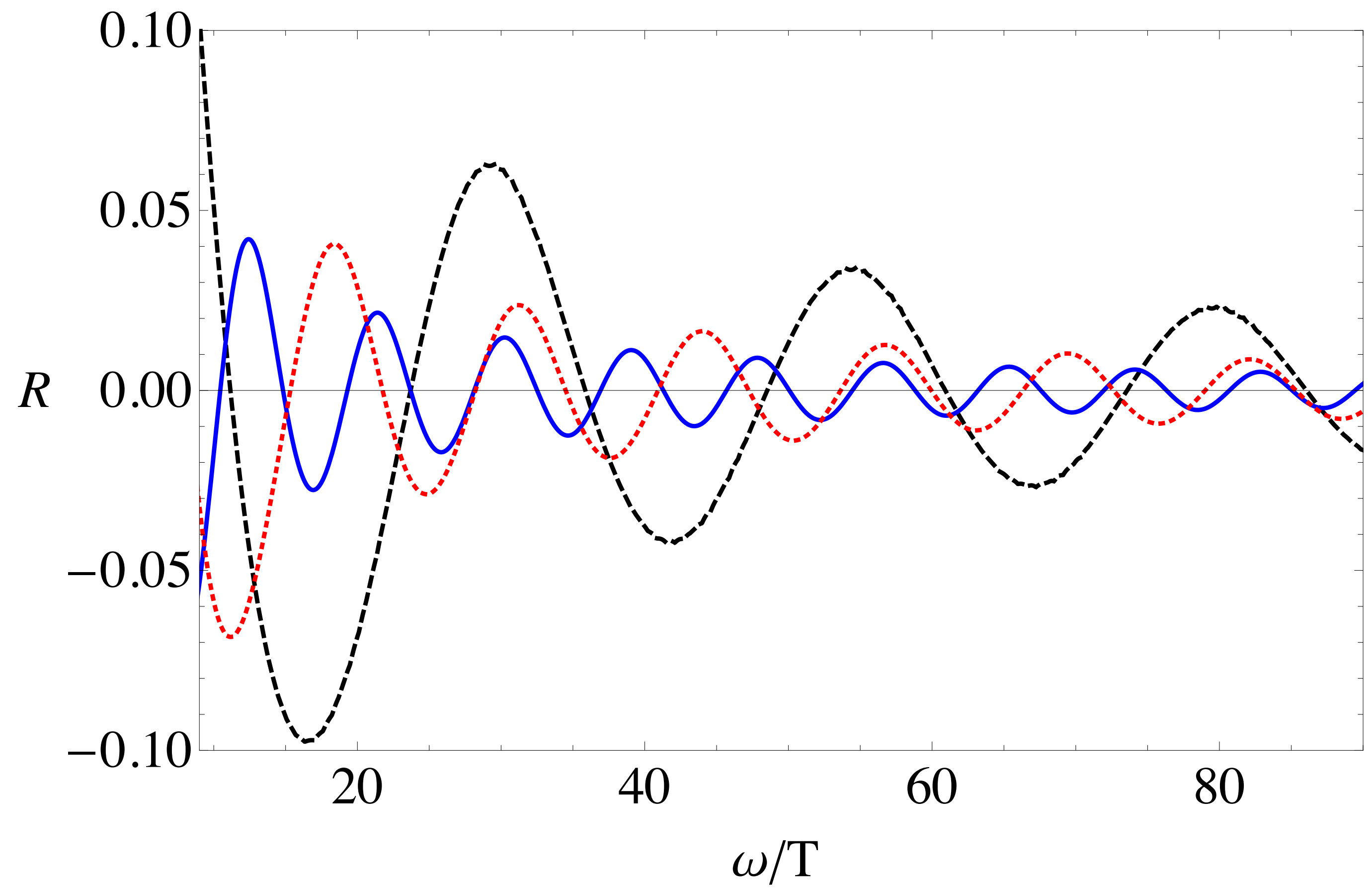
spectral density for for different virtualities

- Out of equilibrium effect: oscillations around thermal value
- As the shell approaches the horizon equilibrium is reached

Relative deviation of spectral density

- Usefull measure of out-of equilibriumness: Relative deviation of spectral density from thermal limit

$$R(\hat{\omega}) = \frac{\chi(\hat{\omega}) - \chi_{th}(\hat{\omega})}{\chi_{th}(\hat{\omega})}$$



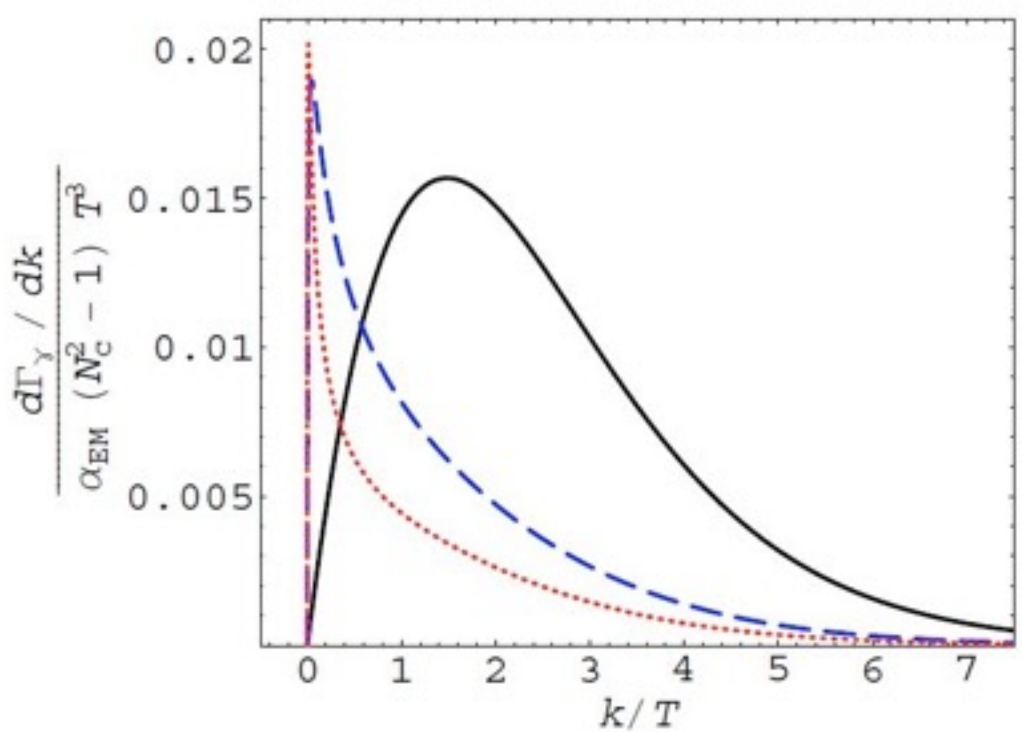
relative deviation R for different virtualities c=1, 0.8, 0

- Top down thermalization: highly energetic modes are closer to equ. value
- Highly virtual field modes thermalize first

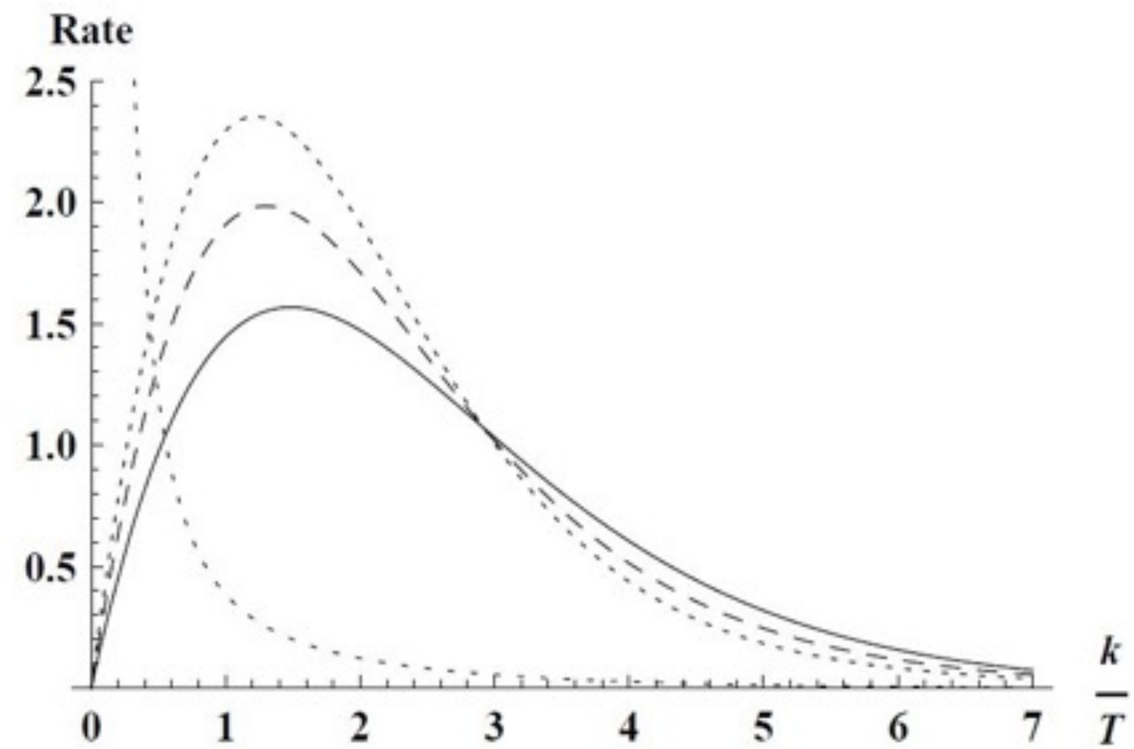
$$\chi(\hat{\omega}) \approx \hat{\omega}^{\frac{2}{3}} \left(1 + \frac{f_1(u_s)}{\hat{\omega}}\right), \qquad R \approx \frac{1}{\hat{\omega}}$$

Photon emission in equilibrium SYM plasma

Photon production



Huot et al (2006)



Hassanain, Schvellinger (2012)

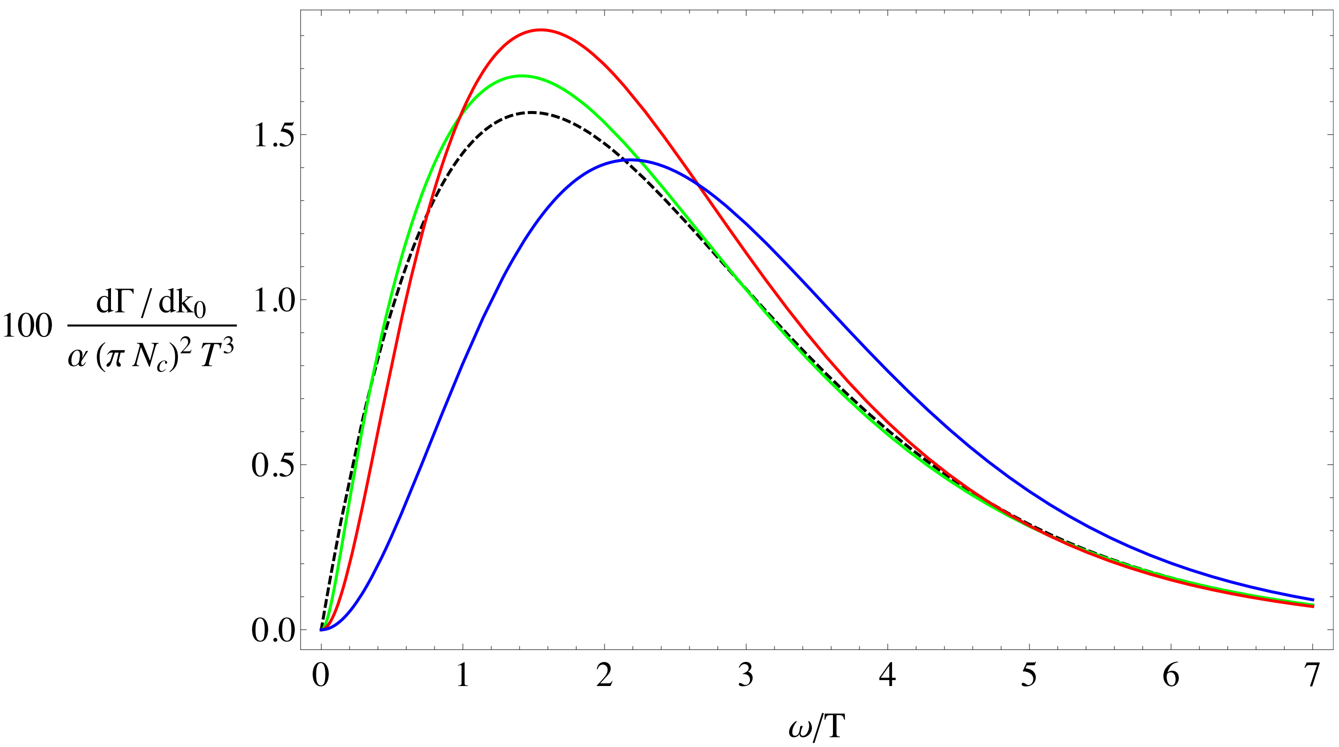
Perturbative result

- Increasing the coupling: slope at k=0 decreases, hydro peak broadens and moves right

Strong coupling result

- Decreasing coupling from $\lambda = \infty$: peak sharpens and moves left

Photon production rate at infinite coupling

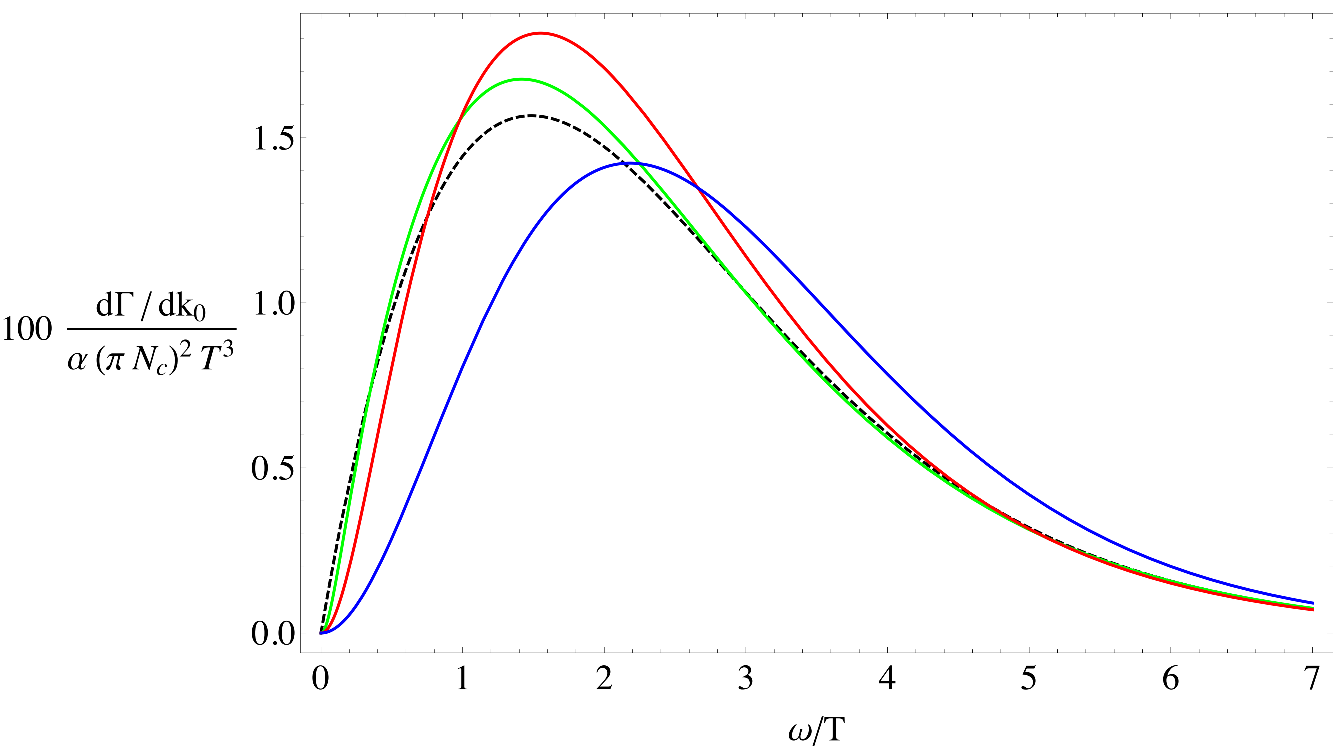


photon production rate

Baier, SS, Taanila, Vuorinen (2012)

- Enhancement of production rate
- Hydro peak broadens and moves right
- Apparently no dramatic observable signature in off-equilibrium photon production

Photon production rate at infinite coupling



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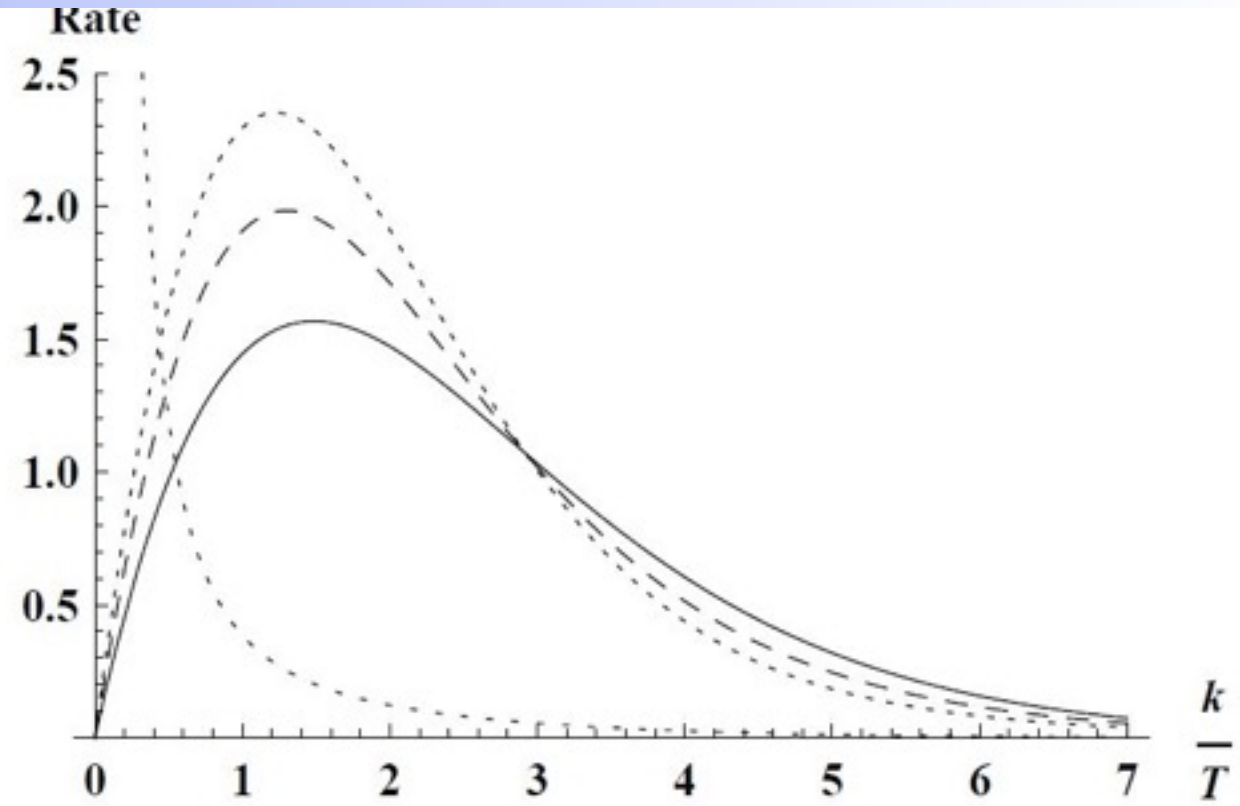
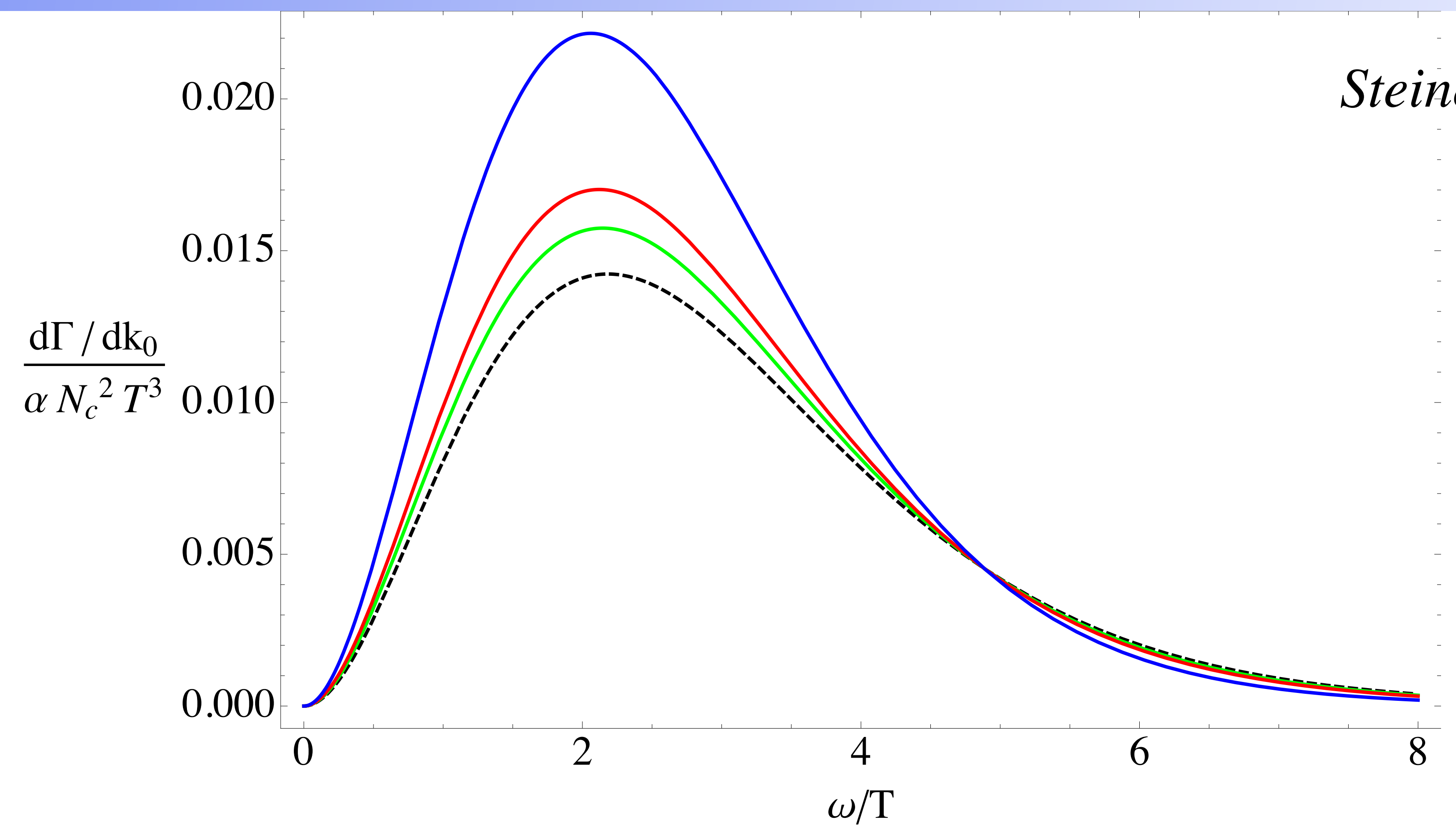


- Enhancement of production rate
- Hydro peak broadens and moves right
- Apparently no dramatic observable signature in off-equilibrium photon production
- Combining the two allows to study thermalization at finite coupling!

Photon production rate at intermediate coupling



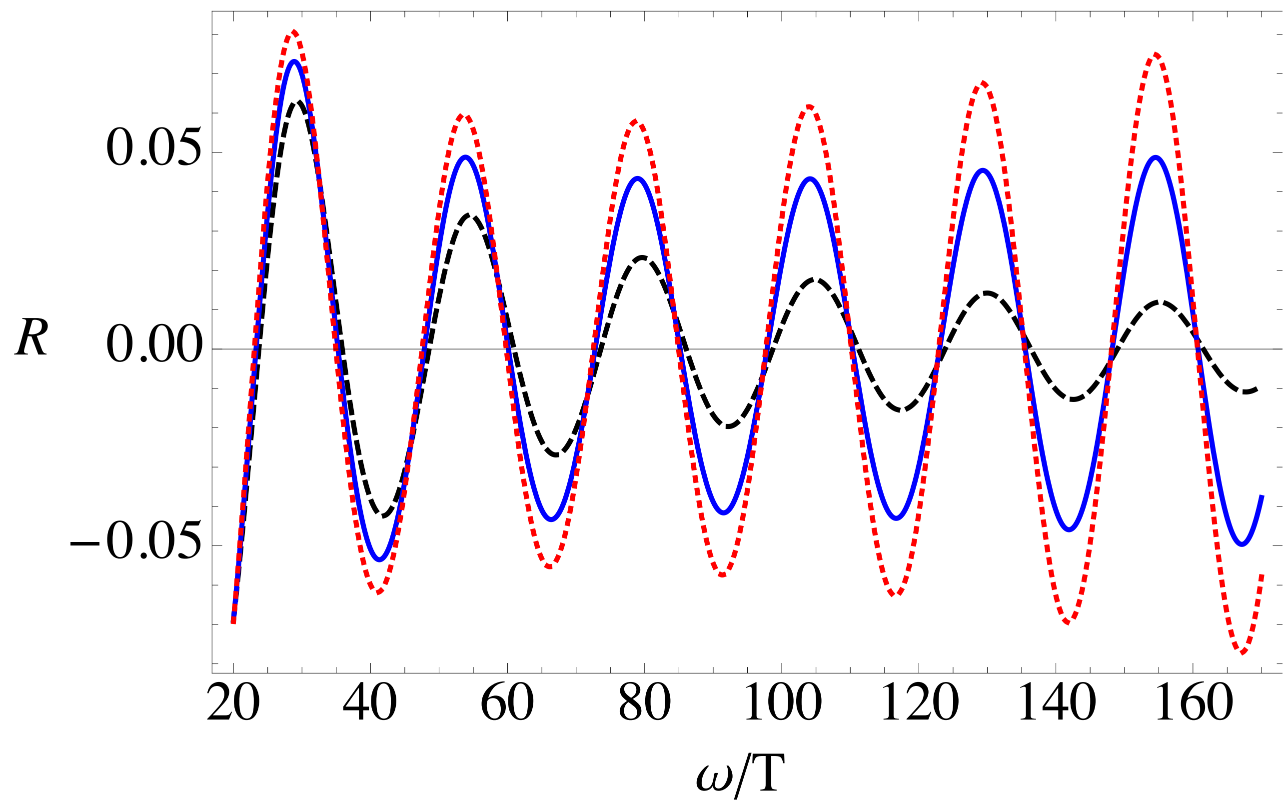
Steineder, SS, Vuorinen (2013)

emission rate for photons r_s/r_h=1.01 and $\lambda = \infty, 120, 80, 40$

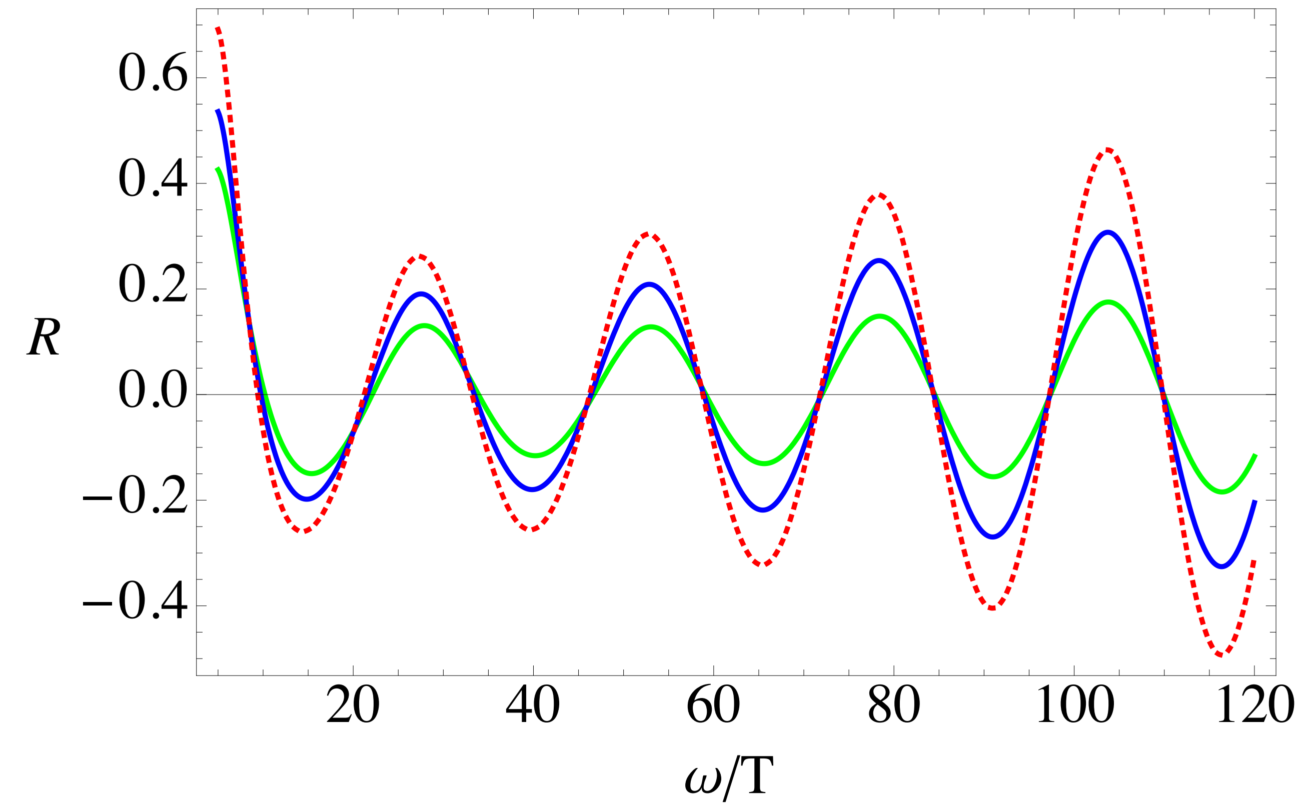
- Behaviour qualitatively similar to equilibrium case: in particular the result is much less sensitive to finite coupling corrections than QNM spectrum

Thermalization at finite coupling

Relative deviation from thermal limit for on shell photons



R for $\lambda = \infty,\ 500,\ 300$



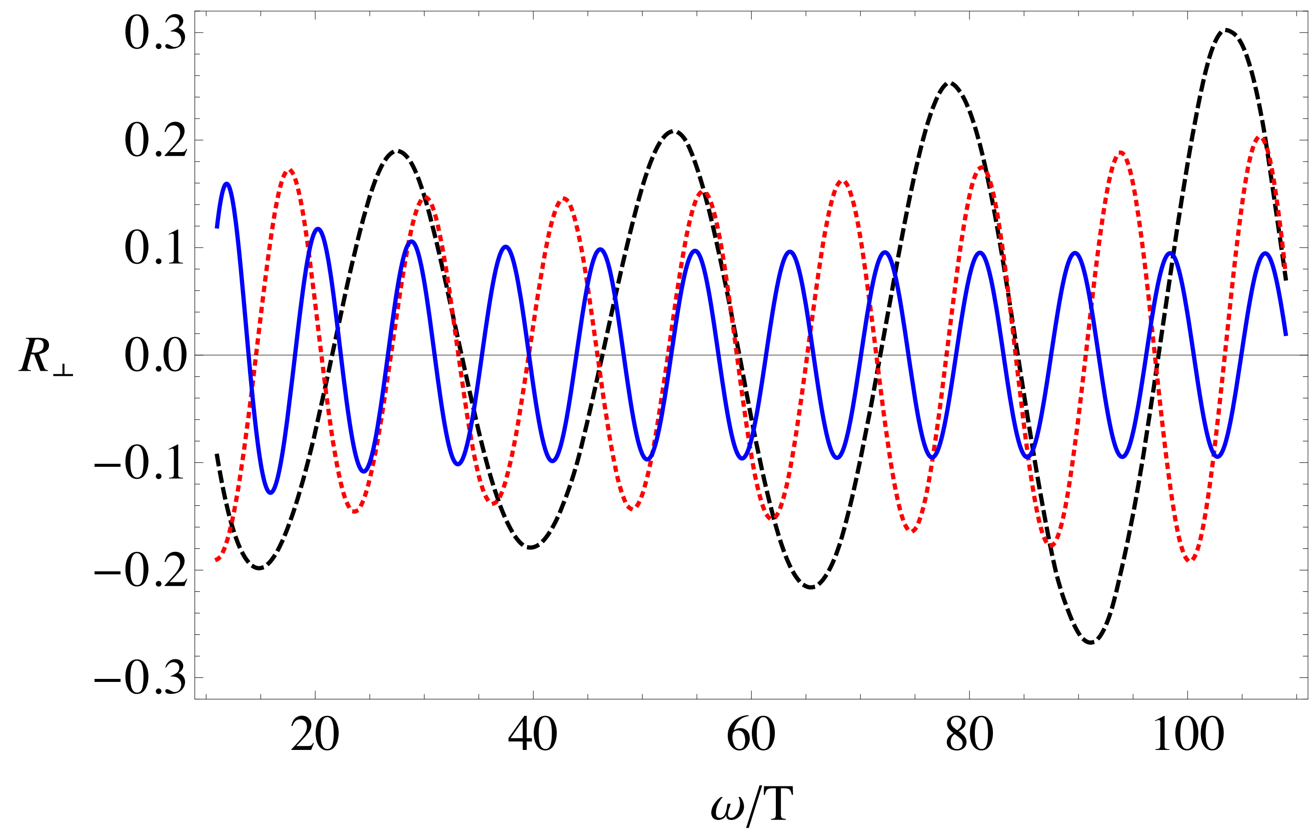
R for $\lambda = 150,\ 100,\ 75$

- Behaviour of relative deviation changes at large frequency
- UV modes are no longer first to thermalize
- Decreasing the coupling: change happens at lower frequency

Thermalization at finite coupling

Virtuality dependence of the relative deviation

R for r_s/r_h=1.1 and c=1, 0.8, 0 for $\lambda = 100$



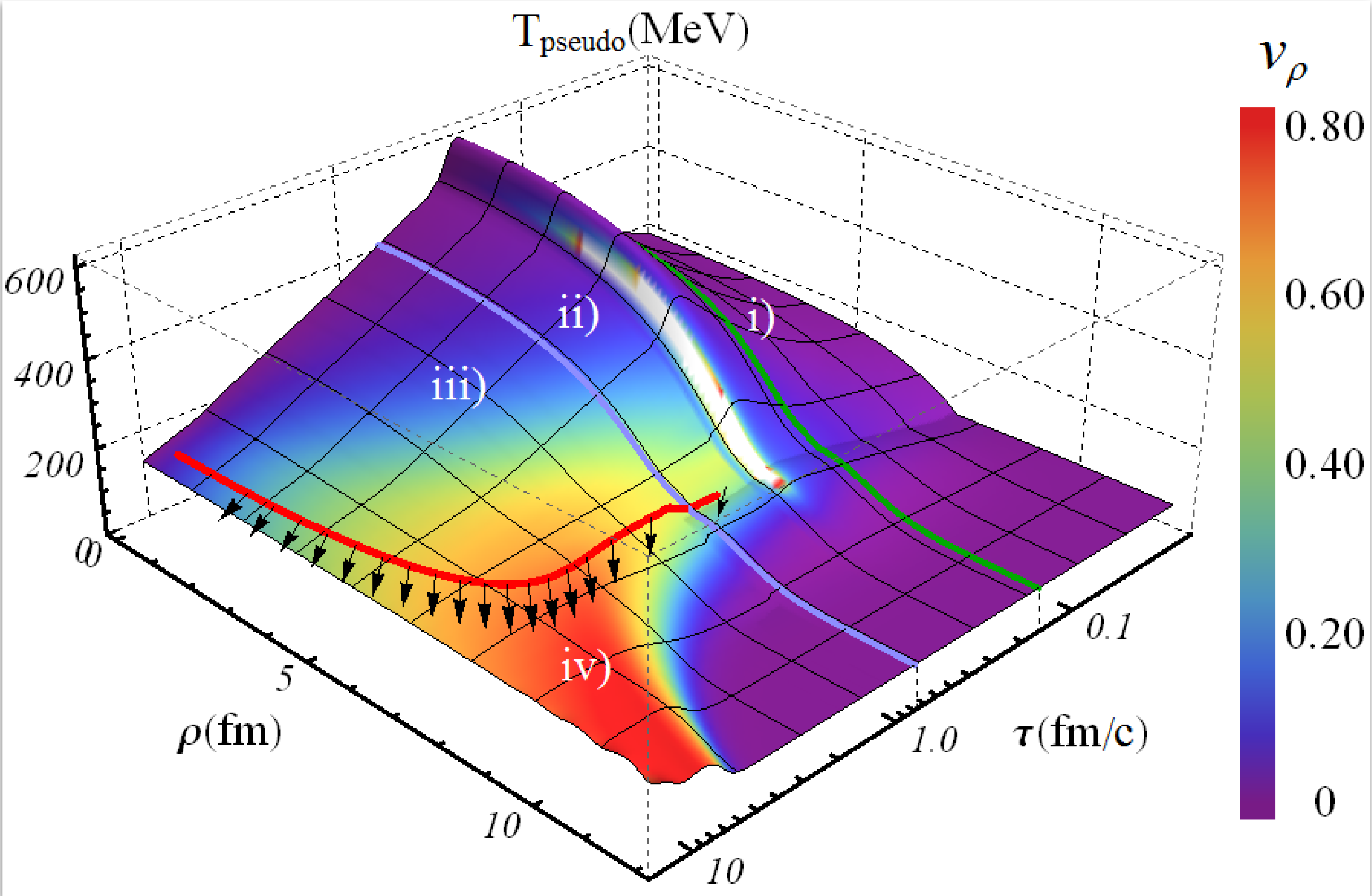
- For maximally virtual photons (c=0), R approaches a constant at $\omega \to \infty$
- For on-shell photons (c=1): amplitude of R rises linearly with ω
- Indication that thermalization pattern changes from top-down towards bottom-up

New developments

Towards the full evolution

combing AdS/CFT with Hydro and kinetic theory

1. Early time expansion
2. AdS/CFT
3. Viscous hydro
4. Kinetic theory



van der Schee, Romatschke, Pratt (2014)

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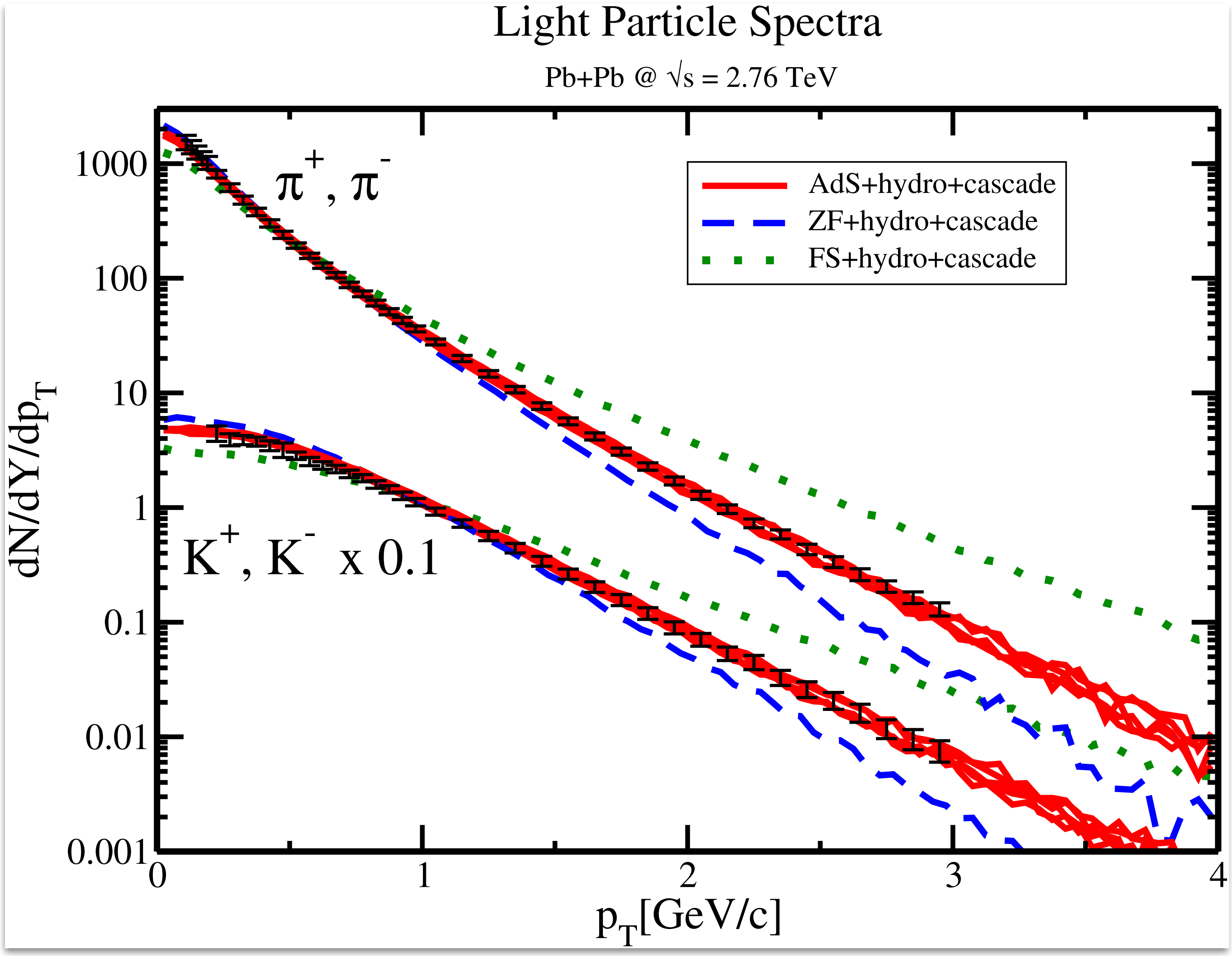
van der Schee, Romatschke, Pratt (2014)



New developments

Hybrid approach

- Combine perturbative field theory with AdS/CFT
- Couple hard and soft sector s.t. they can exchange energy
- Initial conditions are fixed by field theory

Iancu, Mukhopadhyay (2014)

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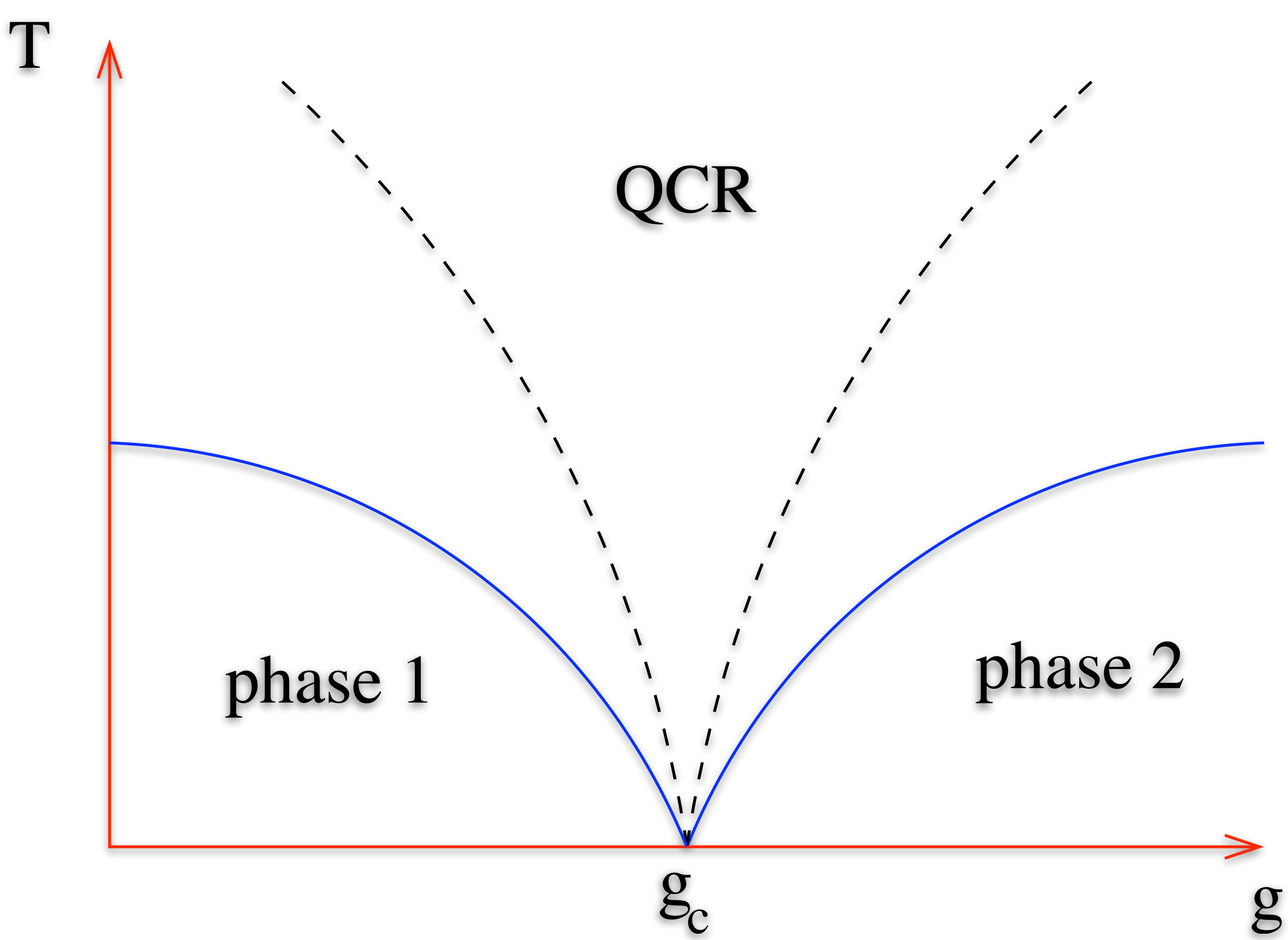
Preliminary results

- Proof of principle: Algorithm converges
- Energy exchange from the hard to the soft sector

Preis, Rebhan, SS

Condensed matter applications

Rather new field in AdS/CFT now termed AdS/CMT



Herzog (2008)

Motivation

- High Tc superconductors and explain the phase diagram
- Phase transition is driven by quantum fluctuations
- QCP are often described by conformal field theories

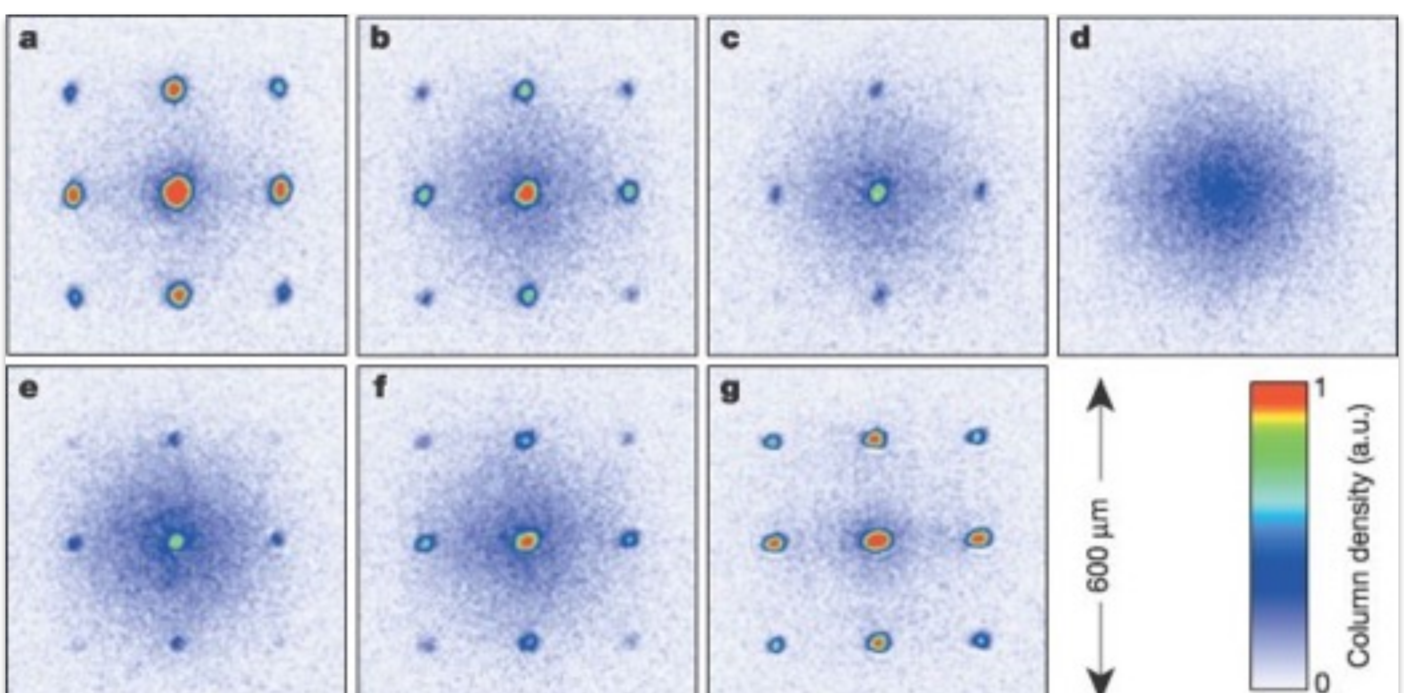
Condensed matter applications

Quantum quenches

- Prepare the system in some ground state and change one parameter instantaneously and let the system evolve
- Does the system thermalize?
- What is the thermalization time?

Useful probes

- Correlation functions
- Entaglement entropy



Quantum revivals, Greiner et al (2002)

Entaglement entropy

Definition

- Split a system into two parts, a subsystem of interest A and the rest B
- Nonlocal quantity
- Observables in A (e.g. correlation functions) are determined by the reduced density matrix

$$\rho_A = \mathrm{Tr}_B \rho$$

- Von Neuman entropy of subsystem A

$$S_{EE}(\rho) = -\mathrm{Tr}_A(\rho_A \log \rho_A)$$

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Properties

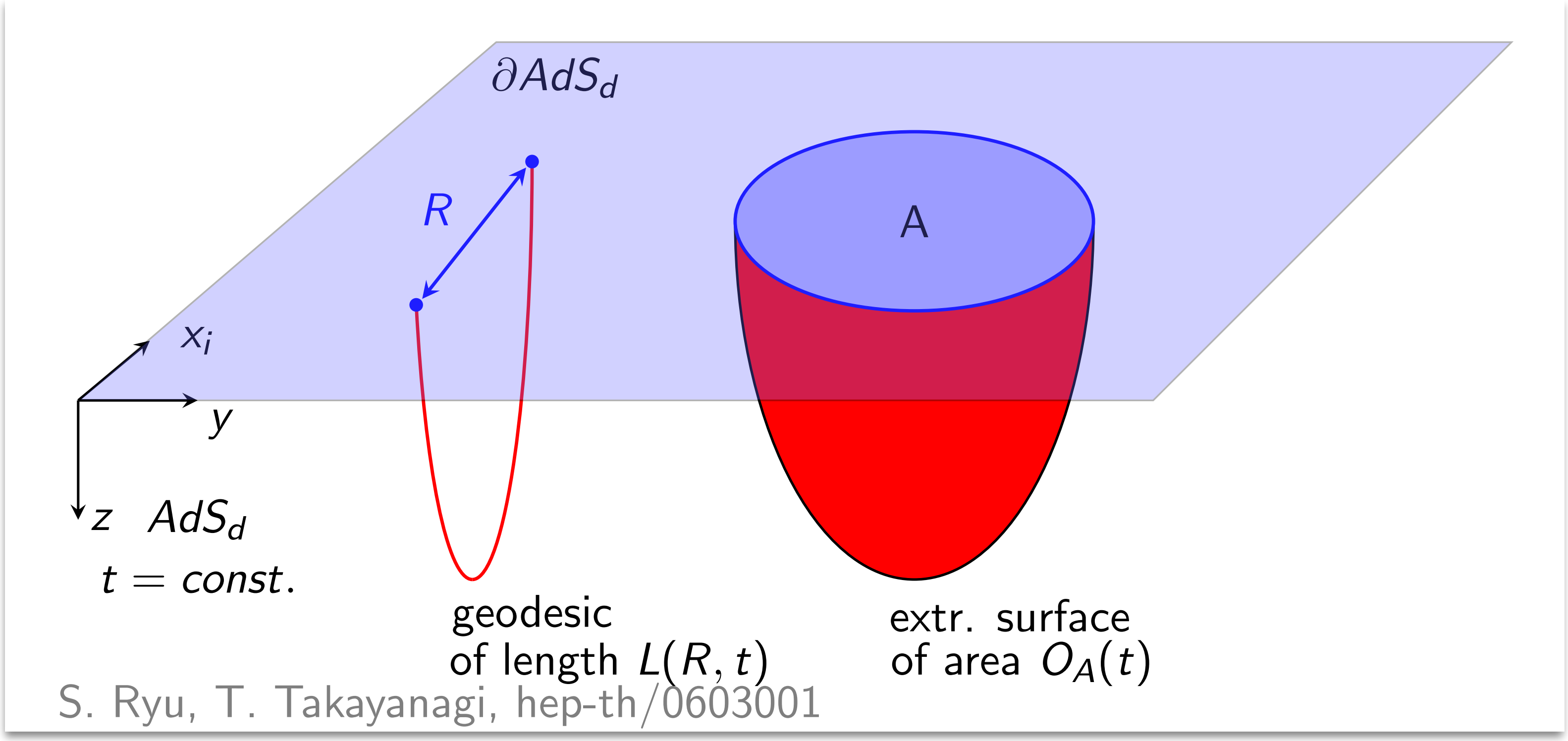
- Serves as an order parameter in condensed matter systems
- Prop. to the degrees of freedom
- measure for quantum information
- Prop. to the area of the entangling surface
- Measure for entropy production in HIC ?

Entaglement entropy

The gravity story

- The EE is conjectured to be given by extremal surfaces

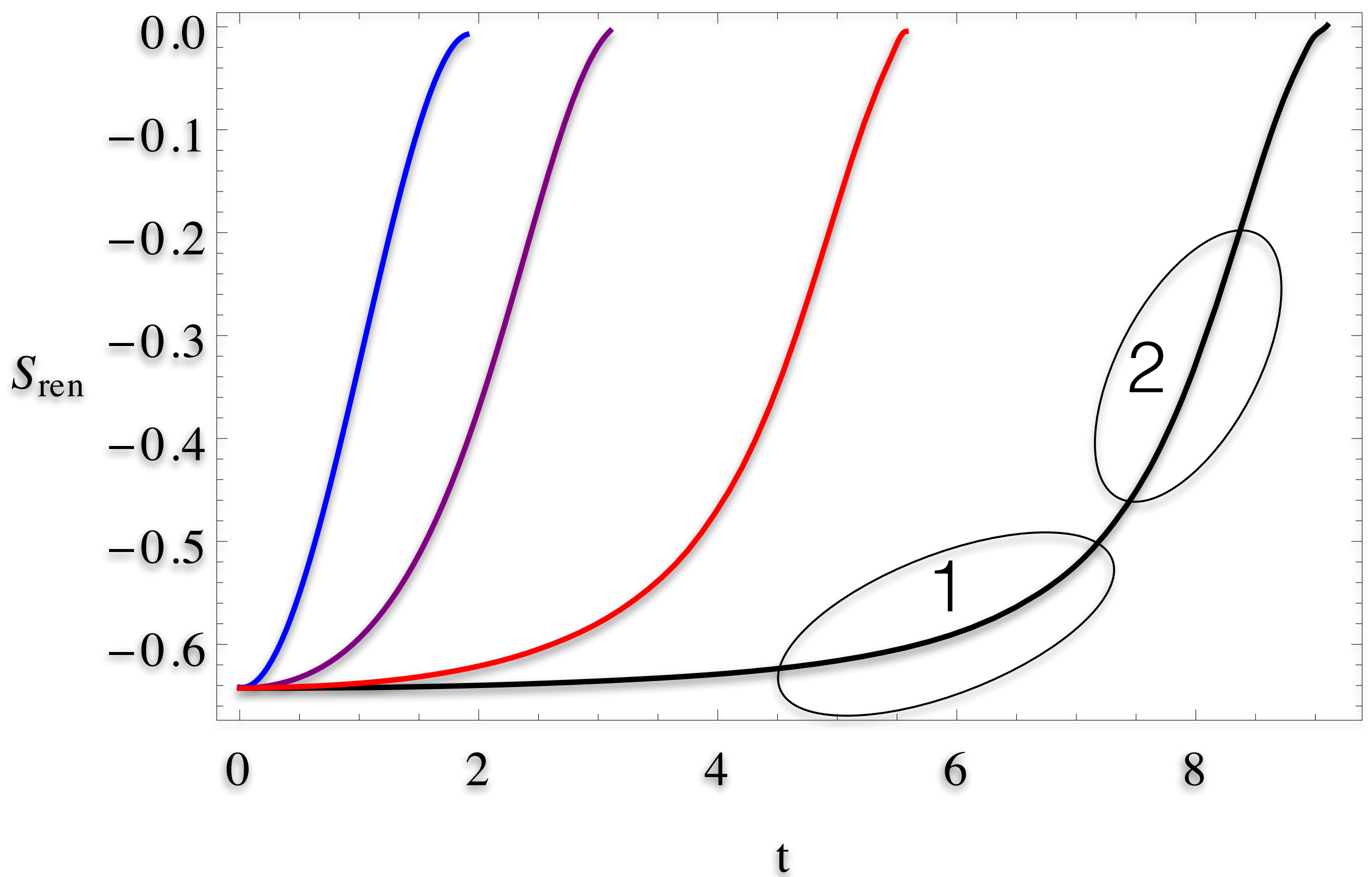
$$S_{EE} = \frac{A_{ext}}{4G_N}$$



- Easy to calculate

EE in the collapsing shell setup

EE for different equations of state p = c E



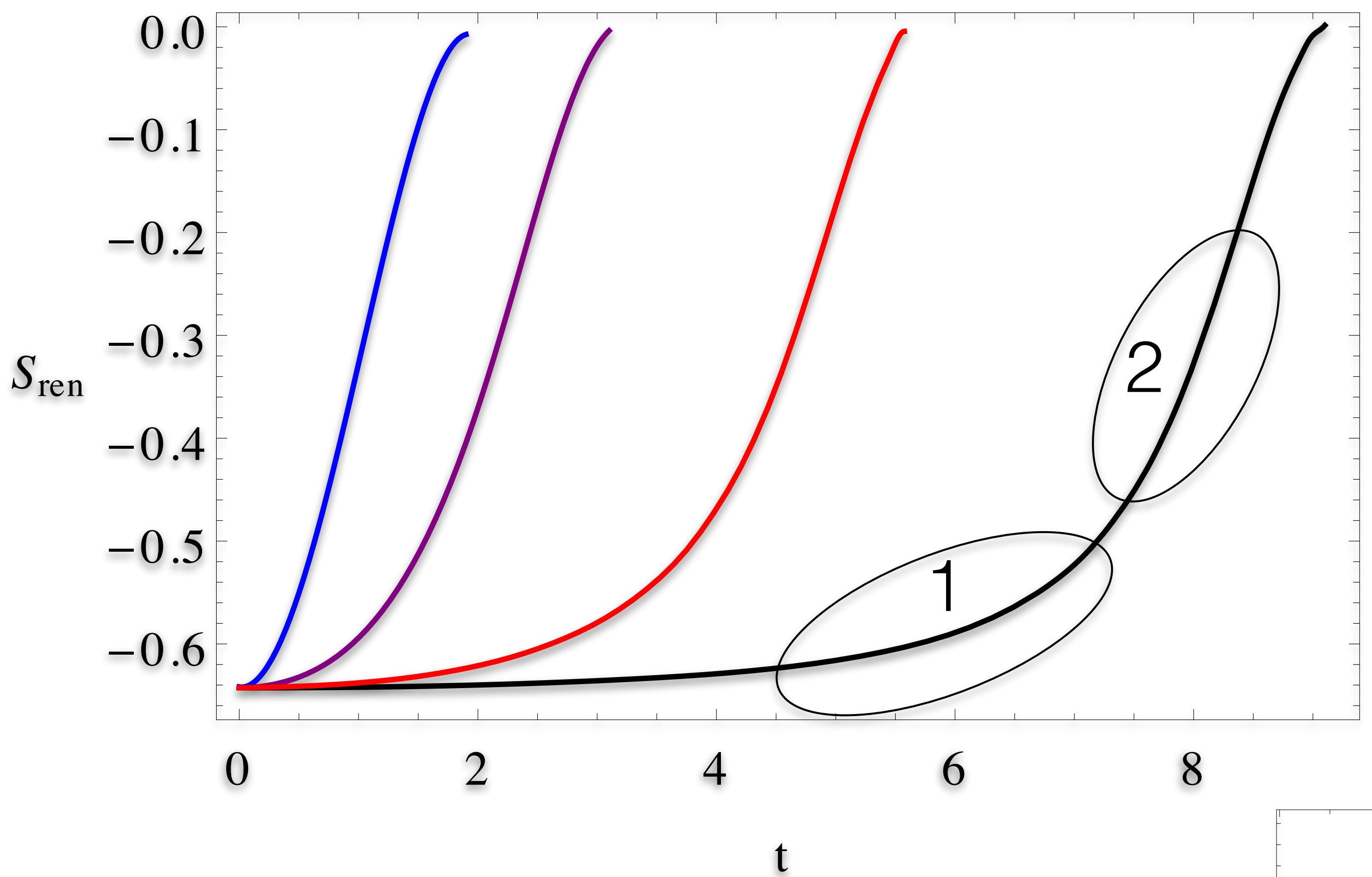
Keranen, Nishimura, SS, Taanila, Vuorinen (2014)

Collapsing shell model serves as a simple toy model for a thermal quench

1. Quadratic part: $S_{ren} = \gamma t^2$
2. Linear part: $S_{ren} = A s_{eq} v_E t$

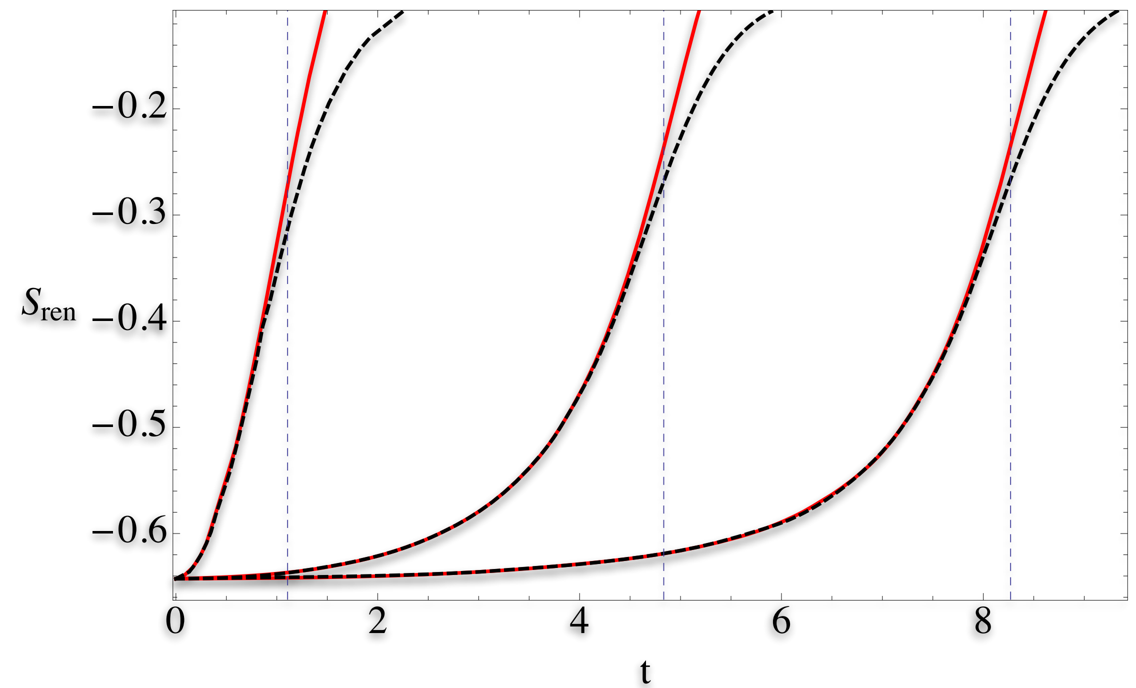
EE in the collapsing shell setup

EE for different equations of state p = c E



Keranen, Nishimura, SS, Taanila, Vuorinen (2014)

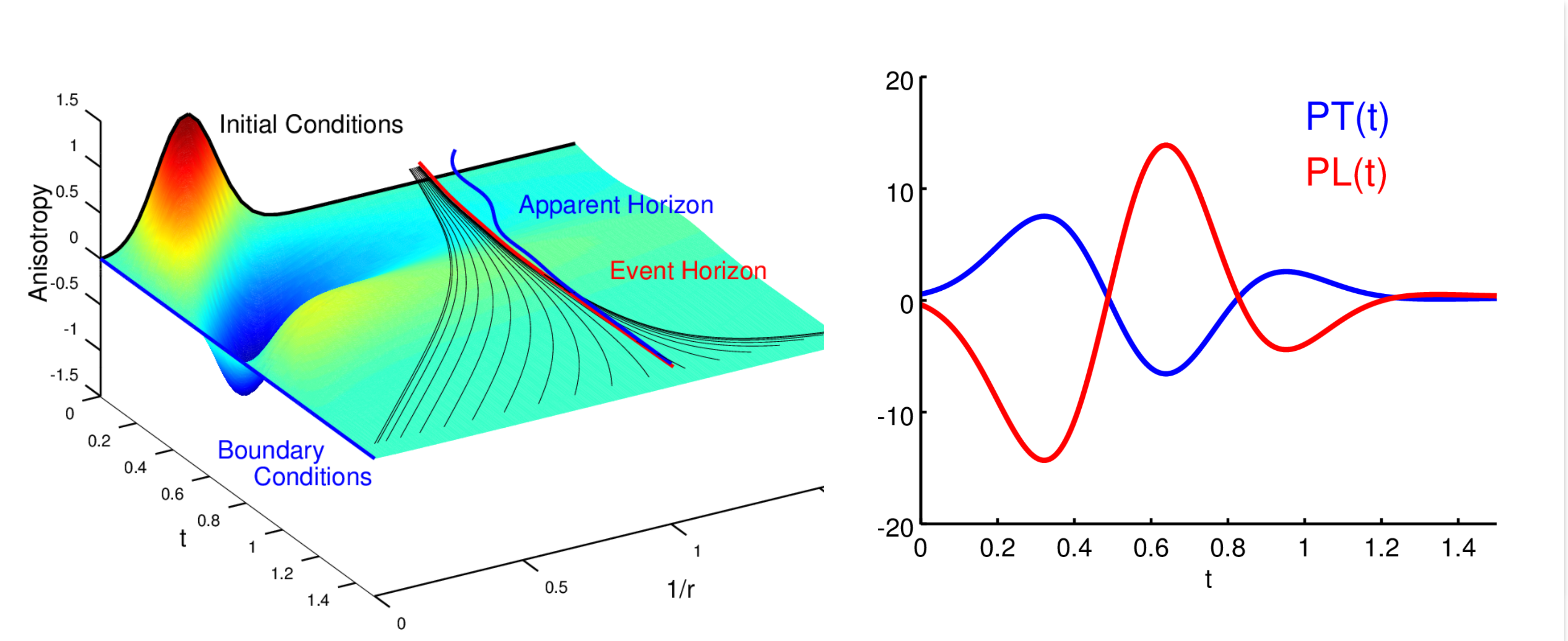
- **Quasistatic approximation:**



EE in anisotropic backgrounds

Towards more realistic problems

- Solve full time dependent Einstein equations



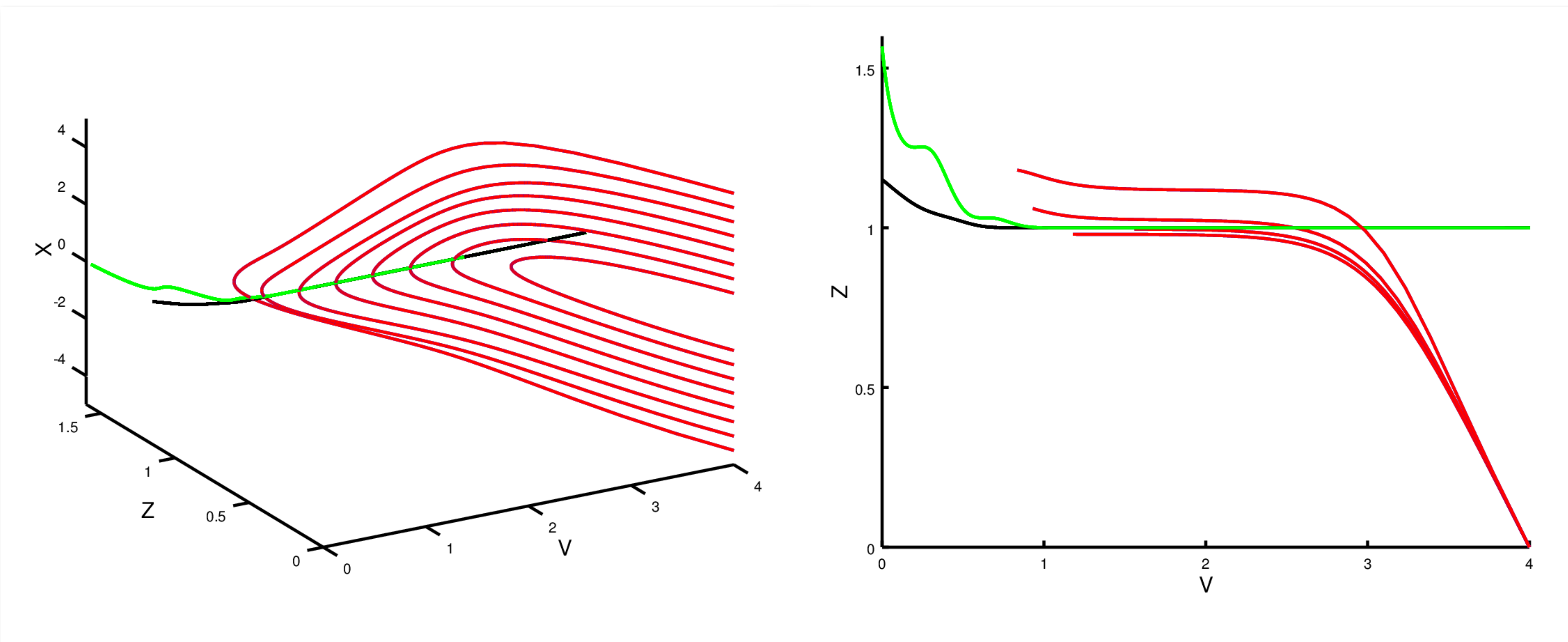
Chesler, Yaffe (2008)

EE in anisotropic backgrounds

Towards more realistic problems

Grumiller, Ecker, SS

- A glance behind the horizon



Conclusions

- AdS/CFT is a very useful playground to study strongly coupled field theories
- Particular useful in studying time dependent problems
- Has already led to fruitful insights
- Only the tip of the iceberg has been touched
- A lot of fun to play around
- Where the field is heading only time and more work can tell